MODELLING THE INPUT SEQUENCING PROBLEM FOR ASSEMBLY FACILITIES

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Abstract **- Mixed model assembly lines are used to produce different product types on the line at the same time. Their correct management requires a number of problems to be solved, among which there is the determination of the input sequence. Traditionally papers appeared in the literature dealt with two objectives, the constant usage of parts on the line and the load levelling of the line. In this paper it is recognized that a different objective is important as well, i.e., the respect of due dates. So, we propose a combined problem, where both the constant usage of parts and the respect of due dates need to be pursued. It is shown how the combined problem can be formulated as an assignment problem, which is known to be an easily solvable problem.**

I. INTRODUCTION

Many companies use mixed-model assembly lines, i.e., lines where different product types are produced on the line at the same time. The effective management of these lines require the solution to several problems, among which there is the determination of the input sequence of products to be assembled on the line.

Traditionally the literature has identified two main objectives to be pursued: the constant rate of usage of parts by the line and the load levelling on each station on the line. In several assembly facilities, however, also another important objective needs to be tackled with in order to manage production flows correctly: the respect of due dates. Take, as an example, the final assembly line of car assembly facilities. This line is in charge of assemblying complex subcomponents (e.g., dashboard, mechanic, etc..) on the main body of the cars. The subcomponents themselves are the final product of other secondary assembly lines. Quality standards and the need to identify precisely all components installed on the car require that a specific coupling is defined between the main body of the car and all its subcomponents. This coupling implies that appointments have to be defined between the main body and each subcomponent. Despite the presence of decoupling buffers at the end of secondary lines, it is still necessary to respect the appointments. Failure to do so results in unnecessary Work In Process, possible overflows of the decoupling buffers, removal of main bodies from the line and, possibly, stops of the whole line.

In this paper, while recognizing the importance of the due date objective, we do not neglect the constant part usage one. The originality of the proposal stands in the fact that it formulates a combined problem, i.e., a problem which considers at the same time the respect of due dates and the constant part usage, as an assignment problem.. It is well known that an assignment problem with $2N$ nodes can be solved in $O(N^3)$ time (see [1]), and that it is one of the most easily solvable problems in the operations research literature. In the existing literature, as it will be better described later, it is possible to find a formulation as an assignment problem only for the constant part usage objective or a combined formulations taking into account the levelling objective and the constant part usage one. This formulation, however, does not show any particularly easy structure and, correspondingly, the authors resort to heuristics to solve the optimization problem.
The remainder of this paper is organized as follows. In Section 2 we briefly review the existing literature. In Section 3 we formulate the problem. Section 4 gives the two cost definitions useful for our formulation, while

Section 5 we formulate the complete problem. Conclusion are drawn in Section 6.

II. LITERATURE REVIEW

After the pioneering work [2] which describes the well-known goal-chasing method to pursue the constant part usage objective, several other works have appeared in the literature concerning the same objective ([3], [4]). The authors in [5] extended the goal-chasing method to multi-level assembly facilities, while the ones in [6] proposed an approach which jointly considers the constant rate and load levelling objectives. A sequencing approach that involves the search for the best permutation of a minimal part set (minimum integer values proportional to periodic production requirements) has been proposed in [7]. The performance of different heuristics approaches to input sequencing on assembly lines with multiple servers versus several performance measures is analyzed in [8]. A different approach to the levelling objective is proposed in [9]. It models the line as a single machine, corresponding to the line's bottleneck. Starting with an integer linear programming formulation aimed at minimizing total tardiness, two different algorithmic approaches, based on cutting planes and branch and bound, are proposed and evaluated.

Recently, two interesting approaches to deal with the constant part usage objective appeared in the literature. The first algorithm ([10]) assigns an ideal due date to each unit of each part type and sums the deviation between these ideal due dates and the actual completion times. Finally, it schedules the products in earliest ideal due date order. Despite the fact that it ignores lower levels in multi-level assembly facilities, the authors report its good performance when compared with the multi-level goal-chasing method. The second approach ([11]) proposes an assignment problem formulation. It evaluates the ideal position of each unit of each product type in the input sequence. Accordingly, it computes a cost for the assignment of these units in positions different from the ideal one. Standing that no-two units can be assigned to the same position in the sequence (and viceversa), it formulates the problem as an assignment one.

III. PROBLEM STATEMENT

Assume that we are considering a manufacturing facility producing m different products at the final level (the mixed-model final assembly line). Suppose that at the final assembly line the total production requirements are $N = \sum_{i=1}^{m} n_i$, where n_i is the production requirement for product type i. The line runs at constant rate and it produces one unit during each time period (or, time frame) k. A due date d_{ij} is associated to each unit j of product i. The objective is the determination of the product to be assembled during each time frame $k = 1,.., n$ so that the production requirements are satisfied for each product, and either the usage goal or the due date goal or both goals are satisfied. It has been shown that the constant usage problem can be formulated as an assignment problem, with proper costs definition.

We show in this paper how to formulate also the due date satisfaction problem as an assignment problem, where the costs are defined with the slack time. The two objectives can, therefore, be pursued together using a weighted sum of the two cost definitions.

IV. THE COST DEFINITIONS

The two main objectives pursued in the input sequencing decision making process are *i*) the respect of due dates and *ii*) the constant part usage on the line.

The first objective, clearly, relates to the ability of the decision maker to satisfy the due date constraints. Note that a job may have accumulated delays in the previous production stages, i.e., it may be not ready for the assembly stages when it should.

The second objective, instead, seeks a constant parts usage on the line. As widely accepted in the literature, this objective is particularly important in *just-in-time* environments. In the following subsections, we describe how to model the problems related to these two different objectives. In the following section, instead, we give a formulation that can be used to take into account both objective at the same time. The overall problem, as we will see, can be formulated as an assignment problem solvable, as known, in polynomial time.

A. *Respect of due dates*

As already mentioned, in this case the decision maker is concerned with the respect of the given due dates assigned to each job requesting to enter the line.

In the following we refer to i, h and k as, respectively, the index for jobs, machines, time intervals. Note that only in this subsection we use the index i to denote a specific job. In all other sections the index i is used to denote a job type. Define p_{ih} as the processing time of job i on machine h and d_i as the due date of job i. If p_i is the total processing time of job i, it holds that

$$p_i = \sum_{h=1}^{H} p_{ih} \qquad (1)$$

The slack time s_i of a job is a measure of its criticality and of its attitude to be late with respect to its due date d_i. If we define TV_i as the total travel time of job i, the slack time of job i if it is assigned at time frame t_k is given by

$$s_i(t_k) = d_i - \sum_{h=1}^{H} p_{ih} - t_k - TV_i \qquad (2)$$

At each instant a possibly different set of jobs is waiting in the input buffer. Let us call this set $W(t_k)$ and its cardinality $w = card\{W(t_k)\}$. The first optimization problem relates to the optimal assignment of waiting jobs to time frames. Assume that our final objective is the maximation of the total or average slack time. The problem is therefore formulated as an assignment problem:

$$\max \quad \sum_{k=1}^{w} \sum_{i=1}^{w} x_{ik}\, s_i(t_k) \qquad (3)$$

$$\text{s.t.} \quad \sum_{k=1}^{w} x_{ik} = 1 \qquad i = 1,..,w \qquad (4)$$

$$\sum_{i=1}^{w} x_{ik} = 1 \qquad k = 1,..,w \qquad (5)$$

$$x_{ik} = \{0,1\} \qquad i,k = 1,..,w$$

Introducing a number S, sufficiently greater than every value that the $s_i(t_k)$ can assume, it is possible to define the values $Sm_i(t_k) = S - s_i(t_k)$. Using these values the objective function (3) can be reformulated as

$$\min \quad \sum_{i=1}^{w} \sum_{k=1}^{w} x_{ik}\, Sm_i(t_k) \qquad (3')$$

B. *Constant usage*

The just-in-time philosophy advocates for a zero-inventory production. It is well known that this objective can be efficiently pursued only if steady demands for products and components occurs. Therefore, the input sequencing in a just-in-time environment has the objective of smoothing the parts usage. Among the several formulations appeared in the literature, we are interested in one ([11]) which shows that the constant usage objectve can be pursued using a formulation of the input sequencing problem as an assignment problem. The authors recognize that, given a product type i whose production demand over the considered time horizon (consisting of n time units) is n_i, the optimal assignment of the n_i unit of i over the n available time frames consists in allowing a constant time interval between any two considered units. This reasoning can be applied separately to every part type i. Unfortunately, the assignment constraints apply, i.e., no two jobs can be assigned to the same time frame and no two time frames can be assigned to the same job. Since it may happen that, under their individual optimal assignment, two different jobs should be assigned to the same time frame, this simple formulation cannot be used. But, however, it is possible to define a cost which penalizes the fact that the j-th job of type i, which should ideally be assigned to the k*-th time frame, is assigned to a different time frame k. Therefore, once defined these costs C^i_{jk}, the authors show how to formulate the constant usage input sequencing problem.

Denoting as C^i_{jk}, x^i_{jk} and I as, respectively, the cost to assign the j-th job of job type i to the k-th time frame, the decision variable that assumes value 1 if the j-th job of job type i to the k-th time frame, and 0 otherwise, and, finally, the set of jobs to be assigned

$I = \{(i,j): i = 1,..,n; j = 1,..,n_i\}$. The problem is formulated as

$$\min \quad \sum_{k=1}^{w} \sum_{(i,j)\in I} C_{jk}^{i} x_{jk}^{i} \qquad (6)$$

$$\text{s.t.} \quad \sum_{k=1}^{w} x_{jk}^{i} = 1 \qquad (i,j) \in I \qquad (7)$$

$$\sum_{(i,j)\in I} x_{jk}^{i} = 1 \quad k = 1,..,w \qquad (8)$$

$$x_{jk}^{i} = \{0,1\} \quad (i,j) \in I; k = 1,..,w$$

For a thorough explanation of the method to compute the costs C_{jk}^{i}, the interested reader can refer to the work mentioned before ([11]).

V. THE COMBINED PROBLEM

In the previous section we have described the two different cost definitions and introduced the corresponding optimization problems. We have shown how these problem can be formulated as assignment problems, with a suitable definitions of the assignment costs. In order to make the two formulations (3)-(5) and (6)-(8) comparable, it is necessary to slightly modify the objective function (3'). In particular, in the internal sum, instead of summing the product units (i.e, over i = 1, .., w), it is possible to sum the product types and the units belonging to each type (i.e., over $(i,j) \in I$), without affecting the meaning of the objective function itself. A slight change has to be introduced also in the notation for the decision variables and the corresponding cost coefficients. Accordingly, it is necessary to modify the constraints (5). So, we reformulate the due date problem as

$$\min \quad \sum_{(i,j)\in I} \sum_{k=1}^{w} x_{jk}^{i} Sm_{j}^{i}(t_k) \qquad (3'')$$

$$\text{s.t.} \quad \sum_{k=1}^{w} x_{jk}^{i} = 1 \qquad (i,j) \in I \qquad (4)$$

$$\sum_{(i,j)\in I} x_{jk}^{i} = 1 \quad k = 1,..,w \qquad (5')$$

$$x_{jk}^{i} = \{0,1\} \quad (i,j) \in I; k = 1,..,w$$

Now, willing to introduce an overall formulation able to consider both objectives, it is simply necessary to introduce suitable weights for the two costs definitions, $Sm_{j}^{i}(t_k)$ and C_{jk}^{i}, namely α_m and α_c. So, the overall optimization problem can be formulated as

$$\min \quad \sum_{k=1}^{w} \sum_{(i,j)\in I} \left(\alpha_m \cdot Sm_{j}^{i}(t_k) + \alpha_c \cdot C_{jk}^{i}\right) x_{jk}^{i} \qquad (9)$$

$$\text{s.t.} \quad \sum_{k=1}^{w} x_{jk}^{i} = 1 \qquad (i,j) \in I \qquad (10)$$

$$\sum_{(i,j)\in I} x_{jk}^{i} = 1 \quad k = 1,..,w \qquad (11)$$

$$x_{jk}^{i} = \{0,1\} \quad (i,j) \in I; k = 1,..,n$$

We have previously assumed that at each instant t_k a possibly different set $W(t_k)$ of jobs is waiting in the input buffers. We denoted with w the cardinality of the set $W(t_k)$. Accordingly, we developed the formulation of the input sequencing problem reported before. Arrivals from the upstream production stages may occur in lots or in single units. In any case, whenever a change occurs in the status of the input buffer, the optimization problem (9) ÷ (11) is solved again. This means that not every assignment determined with the solution of one instance of (9) ÷ (11) is actually the definitive one. On the contrary, several optimization problems are iteratively solved and, at each step, a number of jobs corresponding to the time intervals up to the next (single or bulk) arrival is assigned. In other words, the optimization process is based on a *rolling horizon* perspective. Although one may think this is a local perspective that prevents from obtaining the global optimum, this is exactly what happens in several industrial applications (see for instance the application reported in [12]). The local supervision system, in charge of determining the line input sequence, is not provided with the information on the incoming lots, but its limited time horizon reaches only the jobs currently in the input buffer. In the application mentioned before, this corresponds to a time horizon on roughly 30 minutes.

VI. CONCLUSIONS

In this paper the input sequencing problem for mixed model assembly lines is treated. We recognize that, besides the traditional objectives related to constant part usage and load levelling, also the respect of due dates is a critical issue which needs to be considered. So, we define a combined problem and show how to formulate it as an assignment problem. We believe this formulation is a useful mean to model and properly manage assembly lines.

VII. REFERENCES

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