VOCABULARY

Polyhedron A polyhedron is a solid that is bounded by polygons that enclose a single region of space.

Face The faces of a polyhedron are polygons.

Edge An edge of a polyhedron is a line segment formed by the intersection of two faces.

Vertex A vertex of a polyhedron is a point where three or more edges meet.

Base A base is a polygon that is used to name the polyhedron.

Regular polyhedron A regular polyhedron is a polyhedron whose faces are all congruent regular polygons.

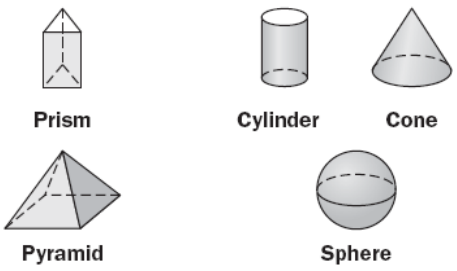
Convex polyhedron A convex polyhedron is a polyhedron such that any two points on its surface can be connected by a line segment that lies entirely inside or on the polyhedron.

Platonic solids A Platonic solid is one of five regular polyhedra: a regular tetrahedron, a cube, a regular octahedron, a regular dodecahedron, and a regular icosahedron.

Cross section A cross section is the intersection of a plane and a solid.

TYPES OF SOLIDS

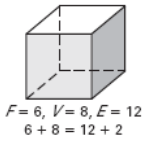
Polyhedra | Not Polyhedra



THEOREM 12.1: EULER'S THEOREM

The number of faces (F), vertices (V), and edges (E) of a polyhedron are related by the formula

$F + V = E +$ 2 .



VOCABULARY

Prism A prism is a polyhedron with two congruent faces, called bases, that lie in parallel planes.

Lateral faces The lateral faces of a prism are parallelograms formed by connecting the corresponding vertices of the bases.

Lateral edges The lateral edges of a prism are the segments connecting the corresponding vertices of the bases.

Surface area The surface area of a polyhedron is the sum of the areas of its faces.

Lateral area The lateral area of a polyhedron is the sum of the areas of its lateral faces.

Net A net of a polyhedron is a two-dimensional representation of the faces of a polyhedron.

Right prism In a right prism, each lateral edge is perpendicular to both bases.

Oblique prism An oblique prism is a prism with lateral edges that are not perpendicular to the bases.

Cylinder A cylinder is a solid with congruent circular bases that lie in parallel planes.

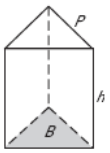
Right cylinder In a right cylinder, the segment joining the centers of the bases is perpendicular to the bases.

THEOREM 12.2: SURFACE AREA OF A RIGHT PRISM

The surface area S of a right prism is

$S = 2B +$ Ph $= aP +$ Ph ,

where a is the apothem of the base, B is the area of a base, P is the perimeter of a base, and h is the height.

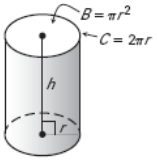


THEOREM 12.3: SURFACE AREA OF A RIGHT CYLINDER

The surface area S of a right cylinder is

$S = 2B + Ch =$ $2\pi r^2 + 2\pi rh$,

where B is the area of a base, C is the circumference of a base, r is the radius of a base, and h is the height.



VOCABULARY

Pyramid A pyramid is a polyhedron in which the base is a polygon and the lateral faces are triangles with a common vertex.

Vertex of a pyramid The vertex of a pyramid is the common vertex of the lateral faces of a pyramid.

Regular pyramid A regular pyramid has a regular polygon for a base and the segment joining the vertex and the center of the base is perpendicular to the base. The lateral faces of a regular pyramid are congruent isosceles triangles.

Slant height The slant height of a regular pyramid is the height of a lateral face of the regular pyramid.

Cone A cone has a circular base and a vertex that is not in the same plane as the base.

Vertex of a cone The vertex of a cone is the point on the cone that is located at a perpendicular distance from the base, called the height of the cone.

Right cone In a right cone, the segment joining the vertex and the center of the base is perpendicular to the base and the slant height is the distance between the vertex and a point on the base edge.

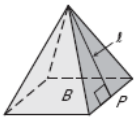
Lateral surface The lateral surface of a cone consists of all segments that connect the vertex with points on the base edge.

THEOREM 12.4: SURFACE AREA OF A REGULAR PYRAMID

The surface area S of a regular pyramid is

$S = B + \frac{1}{2}P\ell$,

where B is the area of the base, P is the perimeter of the base, and ℓ is the slant height.



THEOREM 12.5: SURFACE AREA OF A RIGHT CONE

The surface area S of a right cone is

$S = B + \frac{1}{2}C\ell = \pi r^2 + \pi r\ell$,

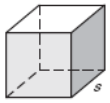
where B is area of the base, C is the circumference of the base, r is the radius of the base, and ℓ is the slant height.

VOCABULARY

Volume The volume of a solid is the number of cubic units contained in its interior.

POSTULATE 27: VOLUME OF A CUBE POSTULATE

The volume of a cube is the cube of the length of its side.



$V = s^3$

POSTULATE 28: VOLUME CONGRUENCE POSTULATE

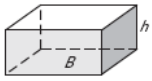
If two polyhedra are congruent, then they have the same volume .

POSTULATE 29: VOLUME ADDITION POSTULATE

The volume of a solid is the sum of the volumes of all its nonoverlapping parts.

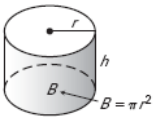
THEOREM 12.6: VOLUME OF A PRISM

The volume V of a prism is $V = Bh$, where B is the area of the base and h is the height.



THEOREM 12.7: VOLUME OF A CYLINDER

The volume V of a cylinder is $V = Bh = \pi r^2 h$, where B is the area of a base, h is the height, and r is the radius of a base.



THEOREM 12.8: CAVALIERI'S PRINCIPLE

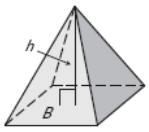
If two solids have the same height and the same cross-sectional area at every level, then they have the same volume .

THEOREM 12.9: VOLUME OF A PYRAMID

The volume V of a pyramid is

$V = \frac{1}{3}Bh$,

where B is the area of the base and h is the height.

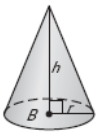


THEOREM 12.10: VOLUME OF A CONE

The volume V of a cone is

$V = \frac{1}{3}Bh = \frac{1}{3}\pi r^2 h$,

where B is area of the base, h is the height, and r is the radius of the base.



VOCABULARY

Sphere A sphere is the set of all points in space equidistant from a given point.

Center of a sphere The center of a sphere is the given point from which all points on the sphere are equidistant.

Radius of a sphere A radius of a sphere is a segment from the center to a point on the sphere.

Chord of a sphere A chord of a sphere is a segment whose endpoints are on the sphere.

Diameter of a sphere A diameter of a sphere is a chord that contains the center of the sphere.

Great circle A great circle is the intersection of a sphere and a plane that contains the center of the sphere.

Hemisphere A hemisphere is one of the congruent halves of a sphere.

THEOREM 12.11: SURFACE AREA OF A SPHERE

The surface area S of a sphere is

$S = 4\pi r^2$,

where r is the radius of the sphere.

THEOREM 12.12: VOLUME OF A SPHERE

The volume V of a sphere is

$V = \frac{4}{3}\pi r^3$,

where r is the radius of the sphere.

VOCABULARY

Similar solids Similar solids are two solids of the same type with equal ratios of corresponding linear measures, such as heights or radii.

THEOREM 12.13: SIMILAR SOLIDS THEOREM

If two similar solids have a scale factor of $a:b$, then corresponding areas have a ratio of $a^2 : b^2$, and corresponding volumes have a ratio of $a^3 : b^3$.