VOCABULARY

Circle A circle is the set of all points in a plane that are equidistant from a given point.

Center The center of a circle is the point from which all points of the circle are equidistant.

Radius A segment from the center of a circle to any point on the circle is a radius.

Chord A chord is a segment whose endpoints are on a circle.

Diameter A diameter is a chord that contains the center of the circle.

Secant A secant is a line that intersects a circle in two points.

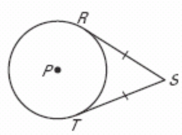
Tangent A tangent is a line in the plane of a circle that intersects the circle in exactly one point.

THEOREM 10.1

In a plane, a line is tangent to a circle if and only if the line is perpendicular to a radius of the circle at its endpoint on the circle.

THEOREM 10.2

Tangent segments from a common external point are congruent.



VOCABULARY

Central angle A central angle of a circle is an angle whose vertex is the center of the circle.

Minor arc Part of a circle measuring less than 180°.

Major arc Part of a circle measuring between 180° and 360°.

Semicircle A semicircle is an arc with endpoints that are the endpoints of a diameter.

Measure of a minor arc The measure of a minor arc is the measure of its central angle.

Measure of a major arc The measure of a major arc is the difference between 360° and the measure of the related minor arc.

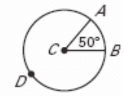
Congruent circles Two circles are congruent circles if they have the same radius.

Congruent arcs Two arcs are congruent arcs if they have the same measure and they are arcs of the same circle or of congruent circles.

MEASURING ARCS

The measure of a minor arc is the measure of its central angle. The expression $m\overset{\frown}{AB}$ is read as "the measure of arc AB."

The measure of the entire circle is 360°. The measure of a major arc is the difference between 360° and the measure of the related minor arc. The measure of a semicircle is 180°.

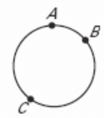


$m\overset{\frown}{AB} = 50°$

$m\overset{\frown}{ADB} = 310°$

POSTULATE 23: ARC ADDITION POSTULATE

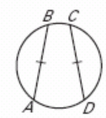
The measure of an arc formed by two adjacent arcs is the sum of the measures of the two arcs.



$m\overset{\frown}{ABC} = m\overset{\frown}{AB} + m\overset{\frown}{BC}$

THEOREM 10.3

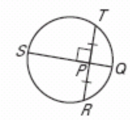
In the same circle, or in congruent circles, two minor arcs are congruent if and only if their corresponding chords are congruent.



$\overset{\frown}{AB} \cong \overset{\frown}{CD}$ if and only if $\overline{AB} \cong \overline{CD}$.

THEOREM 10.4

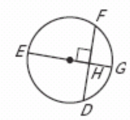
If one chord is a perpendicular bisector of another chord, then the first chord is a diameter.



If $\overline{QS}$ is a perpendicular bisector of $\overline{TR}$, then $\overline{QS}$ is a diameter of the circle.

THEOREM 10.5

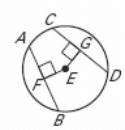
If a diameter of a circle is perpendicular to a chord, then the diameter bisects the chord and its arc.



If $\overline{EG}$ is a diameter and $\overline{EG} \perp \overline{DF}$, then $\overline{HD} \cong \overline{HF}$ and $\overset{\frown}{GD} \cong \overset{\frown}{GF}$.

THEOREM 10.6

In the same circle, or in congruent circles, two chords are congruent if and only if they are equidistant from the center.



$\overline{AB} \cong \overline{CD}$ if and only if $EF = EG$.

VOCABULARY

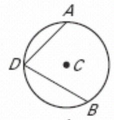
Inscribed angle An inscribed angle is an angle whose vertex is on a circle and whose sides contain chords of the circle.

Intercepted arc The arc that lies in the interior of an inscribed angle and has endpoints on the angle is called the intercepted arc of the angle.

Inscribed polygon A polygon is an inscribed polygon if all of its vertices lie on a circle.

Circumscribed circle A circumscribed circle is a circle that contains the vertices of an inscribed polygon.

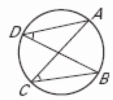
THEOREM 10.7: MEASURE OF AN INSCRIBED ANGLE THEOREM



The measure of an inscribed angle is one half the measure of its intercepted arc.

$m\angle ADB = \frac{1}{2}$ $\underline{m\overset{\frown}{AB}}$

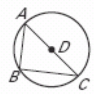
THEOREM 10.8



If two inscribed angles of a circle intercept the same arc, then the angles are congruent.

$\angle ADB \cong \angle$ $\underline{ACB}$

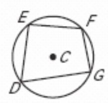
THEOREM 10.9



If a right triangle is inscribed in a circle, then the hypotenuse is a diameter of the circle. Conversely, if one side of an inscribed triangle is a diameter of the circle, then the triangle is a right triangle and the angle opposite the diameter is the right angle.

$m\angle ABC = 90°$ if and only if $\underline{\overline{AC}}$ is a diameter of the circle.

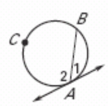
THEOREM 10.10



A quadrilateral can be inscribed in a circle if and only if its opposite angles are supplementary.

D, E, F, and G lie on $\odot C$ if and only if $m\angle D + m\angle F = m\angle E + m\angle G =$ $\underline{180°}$.

THEOREM 10.11

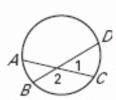


If a tangent and a chord intersect at a point on a circle, then the measure of each angle formed is one half the measure of its intercepted arc.

$m\angle 1 = \frac{1}{2}$ $\underline{m\overset{\frown}{AB}}$

$m\angle 2 = \frac{1}{2}$ $\underline{m\overset{\frown}{BCA}}$

THEOREM 10.12: ANGLES INSIDE THE CIRCLE THEOREM



If two chords intersect *inside* a circle, then the measure of each angle is one half the *sum* of the measures of the arcs intercepted by the angle and its vertical angle.

$m\angle 1 = \frac{1}{2}(m\underline{\overset{\frown}{DC}} + m\underline{\overset{\frown}{AB}})$

$m\angle 2 = \frac{1}{2}(m\underline{\overset{\frown}{AD}} + m\underline{\overset{\frown}{BC}})$

VOCABULARY

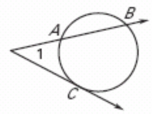
Segments of a chord When two chords intersect in the interior of a circle, each chord is divided into two segments called segments of the chord.

Secant segment A secant segment is a segment that contains a chord of a circle, and has exactly one endpoint outside the circle.

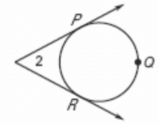
External segment An external segment is the part of a secant segment that is outside the circle.

THEOREM 10.13: ANGLES OUTSIDE THE CIRCLE THEOREM

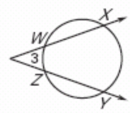
If a tangent and a secant, two tangents, or two secants intersect *outside* a circle, then the measure of the angle formed is one half the *difference* of the measures of the intercepted arcs.



$m\angle 1 = \frac{1}{2}(m\overset{\frown}{BC} - m\overset{\frown}{AC})$



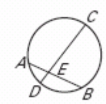
$m\angle 2 = \frac{1}{2}(m\overset{\frown}{PQR} - m\overset{\frown}{PR})$



$m\angle 3 = \frac{1}{2}(m\overset{\frown}{XY} - m\overset{\frown}{WZ})$

THEOREM 10.14: SEGMENTS OF CHORDS THEOREM

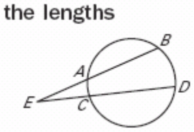
If two chords intersect in the interior of a circle, then the product of the lengths of the segments of one chord is equal to the product of the lengths of the segments of the other chord.



$EA \cdot$ $\underline{EB}$ $= EC \cdot$ $\underline{ED}$

THEOREM 10.15: SEGMENTS OF SECANTS THEOREM

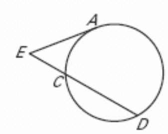
If two secant segments share the same endpoint outside a circle, then the product of the lengths of one secant segment and its external segment equals the product of the lengths of the other secant segment and its external segment.



$EA \cdot$ $\underline{EB}$ $= EC \cdot$ $\underline{ED}$

THEOREM 10.16: SEGMENTS OF SECANTS AND TANGENTS THEOREM

If a secant segment and a tangent segment share an endpoint outside a circle, then the product of the lengths of the secant segment and its external segment equals the square of the length of the tangent segment.



$EA^2 =$ $\underline{EC}$ $\cdot$ $\underline{ED}$

STANDARD EQUATION OF A CIRCLE

The standard equation of a circle with center (h, k) and radius r is:

$(x - h)^2 + (y - k)^2 = r^2$