

**VOCABULARY**

**Image** An image is a new figure produced from the transformation of a figure.

**Preimage** A preimage is the original figure in the transformation of a figure.

**Isometry** An isometry is a transformation that preserves length and angle measure.

**Vector** A vector is a quantity that has both direction and magnitude, or size.

**Initial point** The initial point of a vector is the starting point of the vector.

**Terminal point** The terminal point of a vector is the ending point of the vector.

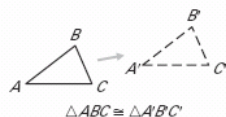
**Horizontal component** The horizontal component describes the left and right direction of a vector.

**Vertical component** The vertical component describes the up and down direction of a vector.

**Component form** The component form of a vector combines the horizontal and vertical components.

**THEOREM 9.1: TRANSLATION THEOREM**

A translation is an isometry.



**VOCABULARY**

**Matrix** A matrix is a rectangular arrangement of numbers in rows and columns.

**Element** Each number in a matrix is called an element.

**Dimensions** The dimensions of a matrix are the numbers of rows and columns.

**VOCABULARY**

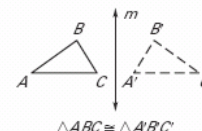
**Line of reflection** In a reflection, the mirror line is called the line of reflection.

**COORDINATE RULES FOR REFLECTIONS**

- If  $(a, b)$  is reflected in the  $x$ -axis, its image is the point  $(a, -b)$ .
- If  $(a, b)$  is reflected in the  $y$ -axis, its image is the point  $(-a, b)$ .
- If  $(a, b)$  is reflected in the line  $y = x$ , its image is the point  $(b, a)$ .
- If  $(a, b)$  is reflected in the line  $y = -x$ , its image is the point  $(-b, -a)$ .

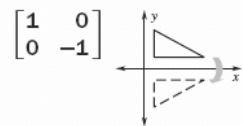
**THEOREM 9.2: REFLECTION THEOREM**

A reflection is an isometry.

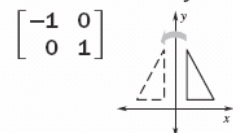


**REFLECTION MATRICES**

Reflection in the  $x$ -axis.



Reflection in the  $y$ -axis.



**VOCABULARY**

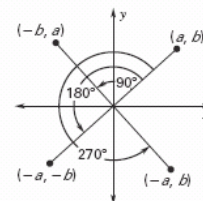
**Center of rotation** In a rotation, a figure is turned about a fixed point called the center of rotation.

**Angle of rotation** In a rotation, rays drawn from the center of rotation to a point and its image form the angle of rotation.

**COORDINATE RULES FOR ROTATIONS ABOUT THE ORIGIN**

When a point  $(a, b)$  is rotated counterclockwise about the origin, the following are true:

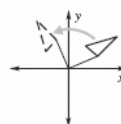
1. For a rotation of  $90^\circ$ ,  $(a, b) \rightarrow (-b, a)$ .
2. For a rotation of  $180^\circ$ ,  $(a, b) \rightarrow (-a, -b)$ .
3. For a rotation of  $270^\circ$ ,  $(a, b) \rightarrow (b, -a)$ .



**ROTATION MATRICES (COUNTERCLOCKWISE)**

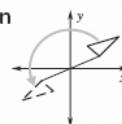
$90^\circ$  rotation

$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$



$180^\circ$  rotation

$$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$



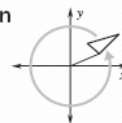
$270^\circ$  rotation

$$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$



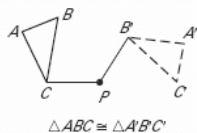
$360^\circ$  rotation

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$



**THEOREM 9.3: ROTATION THEOREM**

A rotation is an isometry.



**VOCABULARY**

**Glide reflection** A glide reflection is a transformation in which every point  $P$  is mapped to a point  $P''$  by the following steps:  
 (1) A translation maps  $P$  onto  $P'$ .  
 (2) A reflection in a line  $k$  parallel to the direction of the translation maps  $P'$  to  $P''$ .

**Composition of transformations** When two or more transformations are combined to form a single transformation, the result is a composition of transformations.

**THEOREM 9.4: COMPOSITION THEOREM**

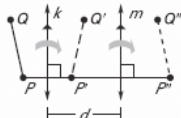
The composition of two (or more) isometries is an isometry.

**THEOREM 9.5: REFLECTIONS IN PARALLEL LINES THEOREM**

If lines  $k$  and  $m$  are parallel, then a reflection in line  $k$  followed by a reflection in line  $m$  is the same as a translation.

If  $P''$  is the image of  $P$ , then:

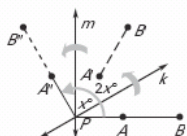
- $\overline{PP''}$  is perpendicular to  $k$  and  $m$ , and
- $PP'' = 2d$ , where  $d$  is the distance between  $k$  and  $m$ .



**THEOREM 9.6: REFLECTIONS IN INTERSECTING LINES THEOREM**

If lines  $k$  and  $m$  intersect at point  $P$ , then a reflection in  $k$  followed by a reflection in  $m$  is the same as a rotation about  $P$ .

The angle of rotation is  $2x^\circ$ , where  $x^\circ$  is the measure of the acute or right angle formed by  $k$  and  $m$ .



**VOCABULARY**

**Line symmetry** A figure in the plane has line symmetry if the figure can be mapped onto itself by a reflection in a line.

**Line of symmetry** In line symmetry, a line of reflection is called a line of symmetry.

**Rotational symmetry** A figure in a plane has rotational symmetry if the figure can be mapped onto itself by a rotation of  $180^\circ$  or less about the center of the figure.

**Center of symmetry** In rotational symmetry, the center of a figure is called the center of symmetry.

**VOCABULARY**

**Scalar multiplication** Scalar multiplication is the process of multiplying each element of a matrix by a real number or *scalar*.