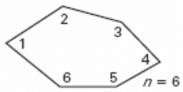
VOCABULARY

Diagonal A diagonal of a polygon is a segment that joins two *nonconsecutive vertices*.

THEOREM 8.1: POLYGON INTERIOR ANGLES THEOREM

The sum of the measures of the interior angles of a convex *n*-gon is $(n - \underline{\ 2\ }) \cdot \underline{\ 180°\ }$.



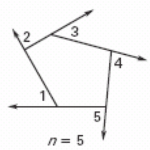
$m\angle 1 + m\angle 2 + \cdots + m\angle n = (n - \underline{\ 2\ }) \cdot \underline{\ 180°\ }$

COROLLARY TO THEOREM 8.1: INTERIOR ANGLES OF A QUADRILATERAL

The sum of the measures of the interior angles of a quadrilateral is $\underline{\ 360°\ }$.

THEOREM 8.2: POLYGON EXTERIOR ANGLES THEOREM

The sum of the measures of the exterior angles of a convex polygon, one angle at each vertex, is $\underline{\ 360°\ }$.



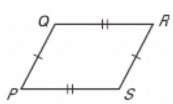
$m\angle 1 + m\angle 2 + \cdots + m\angle n = \underline{\ 360°\ }$

VOCABULARY

Parallelogram A parallelogram is a quadrilateral with both pairs of opposite sides parallel.

THEOREM 8.3

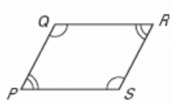
If a quadrilateral is a parallelogram, then its opposite sides are congruent.



If *PQRS* is a parallelogram, then $\underline{\overline{PQ}} \cong \overline{RS}$ and $\overline{QR} \cong \underline{\overline{PS}}$.

THEOREM 8.4

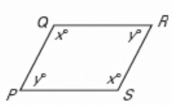
If a quadrilateral is a parallelogram, then its opposite angles are congruent.



If *PQRS* is a parallelogram, then $\angle P \cong \underline{\angle R}$ and $\underline{\angle Q} \cong \angle S$.

THEOREM 8.5

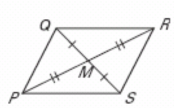
If a quadrilateral is a parallelogram, then its consecutive angles are supplementary.



If *PQRS* is a parallelogram, then $x° + y° = \underline{\ 180°\ }$.

THEOREM 8.6

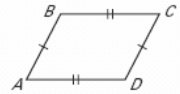
If a quadrilateral is a parallelogram, then its diagonals bisect each other.



$\overline{QM} \cong \underline{\overline{SM}}$ and $\overline{PM} \cong \underline{\overline{RM}}$

THEOREM 8.7

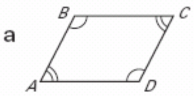
If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.



If $\overline{AB} \cong \underline{\overline{CD}}$ and $\overline{BC} \cong \underline{\overline{AD}}$, then *ABCD* is a parallelogram.

THEOREM 8.8

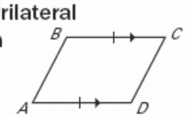
If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.



If $\angle A \cong \underline{\angle C}$ and $\angle B \cong \underline{\angle D}$, then *ABCD* is a parallelogram.

THEOREM 8.9

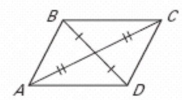
If one pair of opposite sides of a quadrilateral are congruent and parallel, then the quadrilateral is a parallelogram.



If $\overline{BC} \underline{\parallel} \overline{AD}$ and $\overline{BC} \underline{\cong} \overline{AD}$, then *ABCD* is a parallelogram.

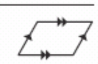
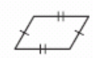
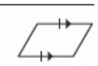
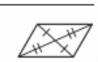
THEOREM 8.10

If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.



If $\overline{BD}$ and $\overline{AC}$ bisect each other, then *ABCD* is a parallelogram.

CONCEPT SUMMARY: WAYS TO PROVE A QUADRILATERAL IS A PARALLELOGRAM

1. Show both pairs of opposite sides are parallel. (Definition) 
2. Show both pairs of opposite sides are congruent. (Theorem 8.7) 
3. Show both pairs of opposite angles are congruent. (Theorem 8.8) 
4. Show one pair of opposite sides are congruent and parallel. (Theorem 8.9) 
5. Show the diagonals bisect each other. (Theorem 8.10) 

VOCABULARY

Rhombus A rhombus is a parallelogram with four congruent sides.

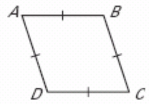
Rectangle A rectangle is a parallelogram with four right angles.

Square A square is a parallelogram with four congruent sides and four right angles.

RHOMBUS COROLLARY

A quadrilateral is a rhombus if and only if it has four congruent sides.

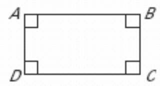
ABCD is a rhombus if and only if $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{AD}$.



RECTANGLE COROLLARY

A quadrilateral is a rectangle if and only if it has four right angles.

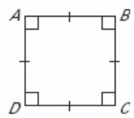
ABCD is a rectangle if and only if $\angle A$, $\angle B$, $\angle C$, and $\angle D$ are right angles.



SQUARE COROLLARY

A quadrilateral is a square if and only if it is a rhombus and a rectangle.

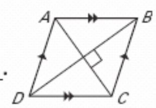
ABCD is a square if and only if $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{AD}$ and $\angle A$, $\angle B$, $\angle C$, and $\angle D$ are right angles.



THEOREM 8.11

A parallelogram is a rhombus if and only if its diagonals are perpendicular.

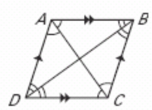
▱*ABCD* is a rhombus if and only if $\overline{AC} \perp \overline{BD}$.



THEOREM 8.12

A parallelogram is a rhombus if and only if each diagonal bisects a pair of opposite angles.

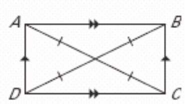
▱*ABCD* is a rhombus if and only if $\overline{AC}$ bisects $\angle BCD$ and $\angle BAD$ and $\overline{BD}$ bisects $\angle ABC$ and $\angle ADC$.



THEOREM 8.13

A parallelogram is a rectangle if and only if its diagonals are congruent.

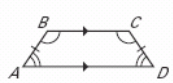
▱*ABCD* is a rectangle if and only if $\overline{AC} \cong \overline{BD}$.



THEOREM 8.14

If a trapezoid is isosceles, then each pair of base angles is congruent.

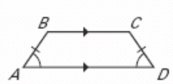
If trapezoid *ABCD* is isosceles, then $\angle A \cong \angle D$ and $\angle B \cong \angle C$.



THEOREM 8.15

If a trapezoid has a pair of congruent base angles, then it is an isosceles trapezoid.

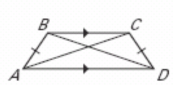
If $\angle A \cong \angle D$ (or if $\angle B \cong \angle C$), then trapezoid *ABCD* is isosceles.



THEOREM 8.16

A trapezoid is isosceles if and only if its diagonals are congruent.

Trapezoid *ABCD* is isosceles if and only if $\overline{AC} \cong \overline{BD}$.



VOCABULARY

Trapezoid A trapezoid is a quadrilateral with exactly one pair of parallel sides.

Bases of a trapezoid The parallel sides of a trapezoid are the bases.

Base angles of a trapezoid A trapezoid has two pairs of base angles. Each pair shares a base as a side.

Legs of a trapezoid The nonparallel sides of a trapezoid are the legs.

Isosceles trapezoid An isosceles trapezoid is a trapezoid in which the legs are congruent.

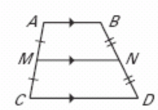
Midsegment of a trapezoid The midsegment of a trapezoid is the segment that connects the midpoints of its legs.

Kite A kite is a quadrilateral that has two pairs of consecutive congruent sides, but opposite sides are not congruent.

THEOREM 8.17: MIDSEGMENT THEOREM FOR TRAPEZOIDS

The midsegment of a trapezoid is parallel to each base and its length is one half the sum of the lengths of the bases.

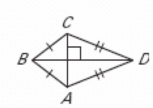
If $\overline{MN}$ is the midsegment of trapezoid *ABCD*, then $\overline{MN} \parallel \overline{AB}$, $\overline{MN} \parallel \overline{DC}$, and $MN = \frac{1}{2}(AB + CD)$.



THEOREM 8.18

If a quadrilateral is a kite, then its diagonals are perpendicular.

If quadrilateral *ABCD* is a kite, then $\overline{AC} \perp \overline{BD}$.



THEOREM 8.19

If a quadrilateral is a kite, then exactly one pair of opposite angles are congruent.

If quadrilateral *ABCD* is a kite and $\overline{BC} \cong \overline{BA}$, then $\angle A \cong \angle C$ and $\angle B \ncong \angle D$.