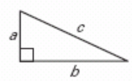
VOCABULARY

Pythagorean triple A Pythagorean triple is a set of three positive integers *a*, *b*, and *c* that satisfy the equation $c^2 = a^2 + b^2$.

THEOREM 7.1: PYTHAGOREAN THEOREM

In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs.



$c^2 = a^2 + b^2$

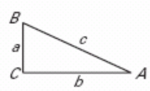
COMMON PYTHAGOREAN TRIPLES AND SOME OF THEIR MULTIPLES

3, 4, 5	**5, 12, 13**	**8, 15, 17**	**7, 24, 25**
6, 8, 10	10, 24, 26	16, 30, 34	14, 48, 50
9, 12, 15	15, 36, 39	24, 45, 51	21, 72, 75
30, 40, 50	50, 120, 130	80, 150, 170	70, 240, 250
$3x, 4x, 5x$	$5x, 12x, 13x$	$8x, 15x, 17x$	$7x, 24x, 25x$

The most common Pythagorean triples are in bold. The other triples are the result of multiplying each integer in a bold face triple by the same factor.

THEOREM 7.2: CONVERSE OF THE PYTHAGOREAN THEOREM

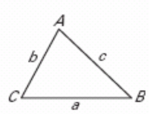
If the square of the length of the longest side of a triangle is equal to the sum of the squares of the lengths of the other two sides, then the triangle is a right triangle.



If $c^2 = a^2 + b^2$, then $\triangle ABC$ is a right triangle.

THEOREM 7.3

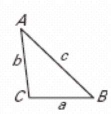
If the square of the length of the longest side of a triangle is less than the sum of the squares of the lengths of the other two sides, then the triangle *ABC* is an acute triangle.



If $c^2 < a^2 + b^2$, then the triangle *ABC* is acute.

THEOREM 7.4

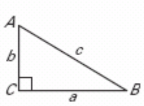
If the square of the length of the longest side of a triangle is greater than the sum of the squares of the lengths of the other two sides, then the triangle *ABC* is an obtuse triangle.



If $c^2 > a^2 + b^2$, then the triangle *ABC* is obtuse.

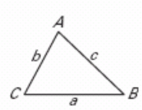
METHODS FOR CLASSIFYING A TRIANGLE BY ANGLES USING ITS SIDE LENGTHS

Theorem 7.2



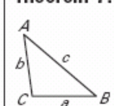
If $c^2 = a^2 + b^2$, then $m\angle C = 90°$ and $\triangle ABC$ is a right triangle.

Theorem 7.3



If $c^2 < a^2 + b^2$, then $m\angle C < 90°$ and $\triangle ABC$ is an acute triangle.

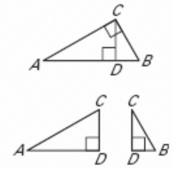
Theorem 7.4



If $c^2 > a^2 + b^2$, then $m\angle C > 90°$ and $\triangle ABC$ is an obtuse triangle.

THEOREM 7.5

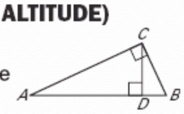
If the altitude is drawn to the hypotenuse of a right triangle, then the two triangles formed are similar to the original triangle and to each other.



$\triangle CBD \sim \triangle ABC$, $\triangle ACD \sim \triangle ABC$, and $\triangle CBD \sim \triangle ACD$.

THEOREM 7.6: GEOMETRIC MEAN (ALTITUDE) THEOREM

In a right triangle, the altitude from the right angle to the hypotenuse divides the hypotenuse into two segments.

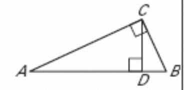


The length of the altitude is the geometric mean of the lengths of the two segments.

$$\frac{BD}{CD} = \frac{CD}{AD}$$

THEOREM 7.7: GEOMETRIC MEAN (LEG) THEOREM

In a right triangle, the altitude from the right angle to the hypotenuse divides the hypotenuse into two segments.

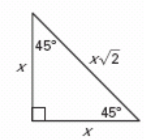


The length of each leg of the right triangle is the geometric mean of the lengths of the hypotenuse and the segment of the hypotenuse that is adjacent to the leg.

$$\frac{AB}{CB} = \frac{CB}{DB} \text{ and } \frac{AB}{AC} = \frac{AC}{AD}$$

THEOREM 7.8: 45°-45°-90° TRIANGLE THEOREM

In a 45°-45°-90° triangle, the hypotenuse is $\sqrt{2}$ times as long as each leg.



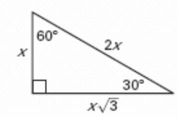
hypotenuse = leg • $\sqrt{2}$

THEOREM 7.9: 30°-60°-90° TRIANGLE THEOREM

In a 30°-60°-90° triangle, the hypotenuse is twice as long as the shorter leg, and the longer leg is $\sqrt{3}$ times as long as the shorter leg.

hypotenuse = 2 • shorter leg

longer leg = shorter leg • $\sqrt{3}$



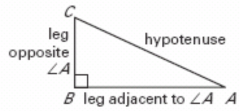
VOCABULARY

Trigonometric ratio A trigonometric ratio is a ratio of the lengths of two sides in a right triangle.

Tangent The ratio of the lengths of the legs in a right triangle is called the tangent of the angle.

TANGENT RATIO

Let $\triangle ABC$ be a right triangle with acute $\angle A$. The tangent of $\angle A$ (written as $\tan A$) is defined as follows:



$$\tan A = \frac{\text{length of leg opposite } \angle A}{\text{length of leg adjacent to } \angle A} = \frac{BC}{AC}$$

VOCABULARY

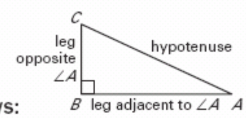
Sine, cosine Sine and cosine are trigonometric ratios for acute angles that involve the lengths of a leg and the hypotenuse of a right triangle.

Angle of elevation When looking up at an object, the angle your line of sight makes with a horizontal line is called the angle of elevation.

Angle of depression When looking down at an object, the angle your line of sight makes with a horizontal line is called the angle of depression.

SINE AND COSINE RATIOS

Let $\triangle ABC$ be a right triangle with acute $\angle A$. The sine of $\angle A$ and cosine of $\angle A$ (written $\sin A$ and $\cos A$) are defined as follows:



$$\sin A = \frac{\text{length of leg opposite } \angle A}{\text{length of hypotenuse}} = \frac{BC}{AC}$$

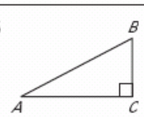
$$\cos A = \frac{\text{length of leg adjacent to } \angle A}{\text{length of hypotenuse}} = \frac{AB}{AC}$$

VOCABULARY

Solve a right triangle To solve a right triangle is to find the measures of all of its sides and angles.

INVERSE TRIGONOMETRIC RATIOS

Let $\angle A$ be an acute angle.



Inverse Tangent If $\tan A = x$, then $\tan^{-1} x = m\angle A$. $\quad \tan^{-1} \frac{BC}{AC} = m\angle A$

Inverse Sine If $\sin A = y$, then $\sin^{-1} y = m\angle A$. $\quad \sin^{-1} \frac{BC}{AB} = m\angle A$

Inverse Cosine If $\cos A = z$, then $\cos^{-1} z = m\angle A$. $\quad \cos^{-1} \frac{AC}{AB} = m\angle A$