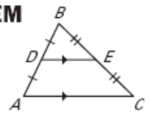
VOCABULARY

Midsegment of a triangle A midsegment of a triangle is a segment that connects the midpoints of two sides of the triangle.

Coordinate proof A coordinate proof involves placing geometric figures in a coordinate plane.

THEOREM 5.1: MIDSEGMENT THEOREM

The segment connecting the midpoints of two sides of a triangle is parallel to the third side and is half as long as that side.



$\overline{DE} \parallel \overline{AC}$ and $DE = \frac{1}{2}AC$

VOCABULARY

Perpendicular bisector A segment, ray, line, or plane that is perpendicular to a segment at its midpoint is called a perpendicular bisector.

Equidistant A point is equidistant from two figures if the point is the *same distance* from each figure.

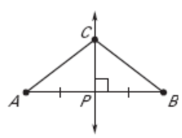
Concurrent When three or more lines, rays, or segments intersect in the same point, they are called concurrent lines, rays, or segments.

Point of concurrency The point of intersection of concurrent lines, rays, or segments is called the point of concurrency.

Circumcenter The point of concurrency of the three perpendicular bisectors of a triangle is called the circumcenter of the triangle.

THEOREM 5.2: PERPENDICULAR BISECTOR THEOREM

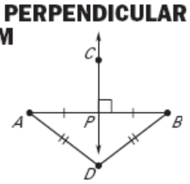
In a plane, if a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment.



If $\overleftrightarrow{CP}$ is the $\perp$ bisector of $\overline{AB}$, then $CA =$ CB.

THEOREM 5.3: CONVERSE OF THE PERPENDICULAR BISECTOR THEOREM

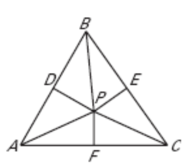
In a plane, if a point is equidistant from the endpoints of a segment, then it is on the perpendicular bisector of the segment.



If $DA = DB$, then D lies on the $\perp$ bisector of $\overline{AB}$.

THEOREM 5.4: CONCURRENCY OF PERPENDICULAR BISECTORS OF A TRIANGLE

The perpendicular bisectors of a triangle intersect at a point that is equidistant from the vertices of the triangle.



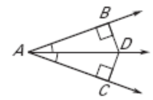
If $\overline{PD}$, $\overline{PE}$, and $\overline{PF}$ are perpendicular bisectors, then $PA =$ PB $=$ PC.

VOCABULARY

Incenter The point of concurrency of the three angle bisectors of a triangle is called the incenter of the triangle.

THEOREM 5.5: ANGLE BISECTOR THEOREM

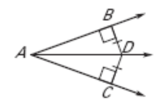
If a point is on the bisector of an angle, then it is equidistant from the two sides of the angle.



If $\overrightarrow{AD}$ bisects $\angle BAC$ and $\overline{DB} \perp \overrightarrow{AB}$ and $\overline{DC} \perp \overrightarrow{AC}$, then $DB =$ DC.

THEOREM 5.6: CONVERSE OF THE ANGLE BISECTOR THEOREM

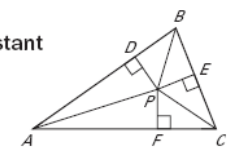
If a point is in the interior of an angle and is equidistant from the sides of the angle, then it lies on the bisector of the angle.



If $\overline{DB} \perp \overrightarrow{AB}$ and $\overline{DC} \perp \overrightarrow{AC}$ and $DB = DC$, then $\overrightarrow{AD}$ bisects $\angle BAC$.

THEOREM 5.7: CONCURRENCY OF ANGLE BISECTORS OF A TRIANGLE

The angle bisectors of a triangle intersect at a point that is equidistant from the sides of the triangle.



If $\overline{AP}$, $\overline{BP}$, and $\overline{CP}$ are angle bisectors of $\triangle ABC$, then $PD =$ PE $=$ PF.

VOCABULARY

Median of a triangle The median of a triangle is a segment from a vertex to the midpoint of the opposite side.

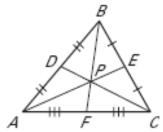
Centroid The point of concurrency of the three medians of a triangle is the centroid.

Altitude of a triangle An altitude of a triangle is the perpendicular segment from a vertex to the opposite side or to the line that contains the opposite side.

Orthocenter The point at which the lines containing the three altitudes of a triangle intersect is called the orthocenter of the triangle.

THEOREM 5.8: CONCURRENCY OF MEDIANS OF A TRIANGLE

The medians of a triangle intersect at a point that is two thirds of the distance from each vertex to the midpoint of the opposite side.

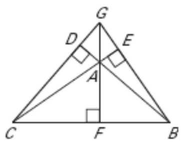


The medians of $\triangle ABC$ meet at P and $AP = \frac{2}{3}$ AE, $BP = \frac{2}{3}$ BF, and $CP = \frac{2}{3}$ CD.

THEOREM 5.9: CONCURRENCY OF ALTITUDES OF A TRIANGLE

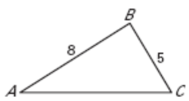
The lines containing the altitudes of a triangle are concurrent.

The lines containing $\overline{AF}$, $\overline{BE}$, and $\overline{CD}$ meet at G.



THEOREM 5.10

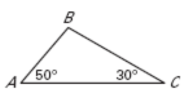
If one side of a triangle is longer than another side, then the angle opposite the longer side is larger than the angle opposite the shorter side.



$AB > BC$, so $m\angle C > m\angle A$.

THEOREM 5.11

If one angle of a triangle is larger than another angle, then the side opposite the larger angle is longer than the side opposite the smaller angle.



$m\angle A > m\angle C$, so $BC > AB$.

THEOREM 5.12: TRIANGLE INEQUALITY THEOREM

The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

$AB + BC > AC$

$AC + BC > AB$

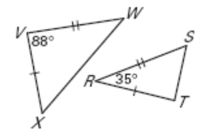
$AB + AC > BC$

VOCABULARY

Indirect Proof An indirect proof uses a temporary assumption that the desired conclusion is false. By then showing that this assumption leads to a logical impossibility, the original statement is proven true *by contradiction*.

THEOREM 5.13: HINGE THEOREM

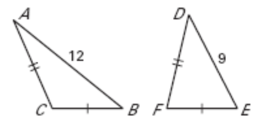
If two sides of one triangle are congruent to two sides of another triangle, and the included angle of the first is larger than the included angle of the second, then the third side of the first is longer than the third side of the second.



$WX > ST$

THEOREM 5.14: CONVERSE OF THE HINGE THEOREM

If two sides of one triangle are congruent to two sides of another triangle, and the third side of the first is longer than the third side of the second, then the included angle of the first is larger than the included angle of the second.



$m\angle C > m\angle F$

HOW TO WRITE AN INDIRECT PROOF

Step 1 **Identify** the statement you want to prove. **Assume** temporarily that this statement is false by assuming that the opposite is true.

Step 2 **Reason** logically until you reach a contradiction.

Step 3 **Point out** that the desired conclusion must be true because the contradiction proves the temporary assumption false.