### VOCABULARY

Amplitude The amplitude of the graph of a sine or cosine function is half the difference of the maximum $M$ and the minimum $m$, or $\frac{1}{2}(M - m)$.

Periodic function A function whose graph has a repeating pattern

Cycle The repeating pattern of a periodic function

Period The horizontal length of a cycle

Frequency The reciprocal of the period; the number of cycles per unit of time

### CHARACTERISTICS OF $y = \sin x$ AND $y = \cos x$

1. The domain of each function is all real numbers.
2. The range of each function is $-1 \le y \le 1$. Therefore, the minimum value of each function is $m = -1$ and the maximum value is $M = 1$.
3. The amplitude of each function's graph is half the difference of the maximum $M$ and the minimum $m$, or $\frac{1}{2}(M - m) = \frac{1}{2}[1 - (-1)] = 1$.
4. Each function is periodic, which means that its graph has a repeating pattern, called a cycle. The horizontal length of each cycle is called the period.
5. The $x$-intercepts of $y =$ sin $x$ occur when $x = 0$, $\pm\pi$, $\pm 2\pi$, $\pm 3\pi$, . . ..
6. The $x$-intercepts of $y =$ cos $x$ occur when $x = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \pm\frac{7\pi}{2}, \ldots$.

### AMPLITUDE AND PERIOD

The amplitude and period of the graphs of $y = a \sin bx$ and $y = a \cos bx$, where $a$ and $b$ are nonzero real numbers, are:

Amplitude $= |a|$ Period $= \frac{2\pi}{|b|}$

### CHARACTERISTICS OF $y = a \tan bx$

The period and vertical asymptotes of the graph of $y = a \tan bx$, where $a$ and $b$ are nonzero real numbers, are:

The period is $\frac{\pi}{|b|}$.

The vertical asymptotes are at odd multiples of $\frac{\pi}{2|b|}$.

### TRANSLATIONS OF SINE AND COSINE GRAPHS

To graph $y = a \sin b(x - h) + k$ or $y = a \cos b(x - h) + k$ where $a > 0$ and $b > 0$, follow these steps:

**Step 1** **Identify** the amplitude $a$, the period $\frac{2\pi}{b}$, the horizontal shift $h$, and the vertical shift $k$ of the graph.

**Step 2** **Draw** the horizontal line $y = k$, called the midline of the graph.

**Step 3** **Find** the five key points by translating the key points of $y = a \sin bx$ or $y = a \cos bx$ horizontally $h$ units and vertically $k$ units.

**Step 4** **Draw** the graph through the five translated key points.

### VOCABULARY

Trigonometric identity A trigonometric equation that is true for all values of $\theta$ (in its domain)

### FUNDAMENTAL TRIGONOMETRIC IDENTITIES

**Reciprocal Identities**

$\csc\theta = \frac{1}{\sin\theta}$ $\sec\theta = \frac{1}{\cos\theta}$ $\cot\theta = \frac{1}{\tan\theta}$

**Tangent and Cotangent Identities**

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\cot\theta = \frac{\cos\theta}{\sin\theta}$

**Pythagorean Identities**

$\sin^2\theta + \cos^2\theta =$ 1

$1 + \tan^2\theta =$ $\sec^2\theta$

$1 + \cot^2\theta =$ $\csc^2\theta$

**Cofunction Identities**

$\sin\left(\frac{\pi}{2} - \theta\right) =$ $\cos\theta$

$\cos\left(\frac{\pi}{2} - \theta\right) =$ $\sin\theta$

$\tan\left(\frac{\pi}{2} - \theta\right) =$ $\cot\theta$

**Negative Angle Identities**

$\sin(-\theta) =$ $-\sin\theta$

$\cos(-\theta) =$ $\cos\theta$

$\tan(-\theta) =$ $-\tan\theta$

### VOCABULARY

Sinusoids Graphs of sine and cosine functions

## SUM AND DIFFERENCE FORMULAS

**Sum Formulas**

$\sin(a + b) = \sin a \cos b + \cos a \sin b$

$\cos(a + b) = \cos a \cos b - \sin a \sin b$

$\tan(a + b) = \dfrac{\tan a + \tan b}{1 - \tan a \tan b}$

**Difference Formulas**

$\sin(a - b) = \sin a \cos b - \cos a \sin b$

$\cos(a - b) = \cos a \cos b + \sin a \sin b$

$\tan(a - b) = \dfrac{\tan a - \tan b}{1 + \tan a \tan b}$

## DOUBLE-ANGLE AND HALF-ANGLE FORMULAS

**Double-Angle Formulas**

$\cos 2a = \cos^2 a - \sin^2 a$

$\cos 2a = 2\cos^2 a - 1$

$\cos 2a = 1 - 2\sin^2 a$

$\sin 2a = 2 \sin a \cos a$

$\tan 2a = \dfrac{2 \tan a}{1 - \tan^2 a}$

**Half-Angle Formulas**

$\sin \dfrac{a}{2} = \pm\sqrt{\dfrac{1 - \cos a}{2}}$

$\cos \dfrac{a}{2} = \pm\sqrt{\dfrac{1 + \cos a}{2}}$

$\tan \dfrac{a}{2} = \dfrac{1 - \cos a}{\sin a}$

$\tan \dfrac{a}{2} = \dfrac{\sin a}{1 + \cos a}$