RIGHT TRIANGLE DEFINITIONS OF TRIGONOMETRIC FUNCTIONS

Let θ be an acute angle of a right triangle. The six trigonometric functions of θ are defined as follows:

$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$ $\qquad \csc\theta = \frac{\text{hypotenuse}}{\text{opposite}}$

$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$ $\qquad \sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}}$

$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$ $\qquad \cot\theta = \frac{\text{adjacent}}{\text{opposite}}$

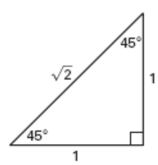
The abbreviations *opp*, *adj*, and *hyp* are often used to represent the side lengths of the right triangle. Note that the ratios in the second column are reciprocals of the ratios in the first column:

$\csc\theta = \frac{1}{\sin\theta}$ $\qquad \sec\theta = \frac{1}{\cos\theta}$ $\qquad \cot\theta = \frac{1}{\tan\theta}$

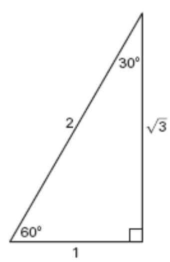
TRIGONOMETRIC VALUES FOR SPECIAL ANGLES

The table below gives the values of the six trigonometric functions for the angles 30°, 45°, and 60°. You can obtain these values from the triangles shown.

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$

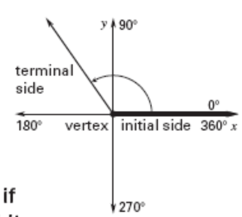


θ	$\csc\theta$	$\sec\theta$	$\cot\theta$
30°	2	$\frac{2\sqrt{3}}{3}$	$\sqrt{3}$
45°	$\sqrt{2}$	$\sqrt{2}$	1
60°	$\frac{2\sqrt{3}}{3}$	2	$\frac{\sqrt{3}}{3}$



ANGLES IN STANDARD POSITION

In a coordinate plane, an angle can be formed by fixing one ray, called the initial side, and rotating the other ray, called the terminal side, about the vertex.



An angle is in standard position if its vertex is at the origin and its initial side lies on the positive x-axis.

VOCABULARY

Initial side and terminal side An angle can be formed by fixing one ray, called the initial side, and rotating the other ray, called the terminal side, about the vertex.

Standard position The position of an angle whose vertex is at the origin and its initial side lies on the positive x-axis

Coterminal Angles in standard position whose terminal sides coincide

Radian In a circle with radius r centered at the origin, one radian is the measure of an angle in standard position whose terminal side intercepts an arc of length r.

Sector A region of a circle that is bounded by two radii and an arc of the circle

Central angle The angle formed by two radii of a circle

CONVERTING BETWEEN DEGREES AND RADIANS

Degrees to radians

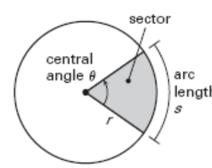
Multiply degree measure by $\frac{\pi \text{ radians}}{180°}$.

Radians to Degrees

Multiply radian measure by $\frac{180°}{\pi \text{ radians}}$.

ARC LENGTH AND AREA OF A SECTOR

The arc length s and area A of a sector with radius r and central angle θ (measured in radians) are as follows.



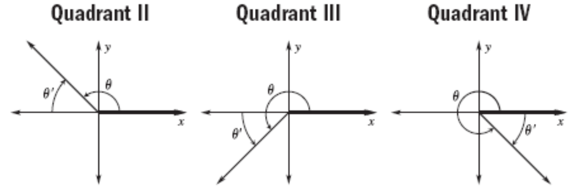
Arc length: $s = r\theta$

Area: $A = \frac{1}{2}r^2\theta$

REFERENCE ANGLE RELATIONSHIPS

Let θ be an angle in standard position. The reference angle for θ is the acute angle θ' formed by the terminal side of θ and the x-axis. The relationship between θ and θ' is shown below for nonquadrantal angles θ such that $90° < \theta < 360°\left(\frac{\pi}{2} < \theta < 2\pi\right)$.

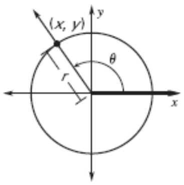
Quadrant II **Quadrant III** **Quadrant IV**



	Quadrant II	Quadrant III	Quadrant IV
Degrees:	$\theta' = 180° - \theta$	$\theta' = \theta - 180°$	$\theta' = 360° - \theta$
Radians:	$\theta' = \pi - \theta$	$\theta' = \theta - \pi$	$\theta' = 2\pi - \theta$

GENERAL DEFINITIONS OF TRIGONOMETRIC FUNCTIONS

Let θ be an angle in standard position, and let (x, y) be the point where the terminal side of θ intersects the circle $x^2 + y^2 = r^2$. The six trigonometric functions of θ are defined as follows:



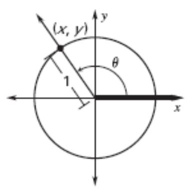
$\sin\theta = \dfrac{y}{r}$ $\csc\theta = \dfrac{r}{y},\ y \neq 0$

$\cos\theta = \dfrac{x}{r}$ $\sec\theta = \dfrac{r}{x},\ x \neq 0$

$\tan\theta = \dfrac{y}{x},\ x \neq 0$ $\cot\theta = \dfrac{x}{y},\ y \neq 0$

THE UNIT CIRCLE

The circle $x^2 + y^2 = 1$, which has center $(0, 0)$ and radius 1, is called the unit circle.



$\sin\theta = \dfrac{y}{r} = \dfrac{y}{1} = \underline{y}$

$\cos\theta = \dfrac{x}{r} = \dfrac{x}{1} = \underline{x}$

EVALUATING TRIGONOMETRIC FUNCTIONS

Use these steps to evaluate a trigonometric function for any angle θ:

STEP 1 Find the reference angle $\underline{\theta'}$.

STEP 2 Evaluate the trigonometric functions for $\underline{\theta'}$.

STEP 3 Determine the sign of the trigonometric function value from the quadrant in which $\underline{\theta}$ lies.

Signs of Function Values

Quadrant II	Quadrant I
$\sin\theta$, $\csc\theta$: +	$\sin\theta$, $\csc\theta$: +
$\cos\theta$, $\sec\theta$: −	$\cos\theta$, $\sec\theta$: +
$\tan\theta$, $\cot\theta$: −	$\tan\theta$, $\cot\theta$: +
Quadrant III	**Quadrant IV**
$\sin\theta$, $\csc\theta$: −	$\sin\theta$, $\csc\theta$: −
$\cos\theta$, $\sec\theta$: −	$\cos\theta$, $\sec\theta$: +
$\tan\theta$, $\cot\theta$: +	$\tan\theta$, $\cot\theta$: −

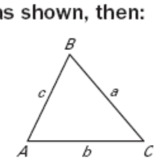
LAW OF COSINES

If $\triangle ABC$ has sides of length a, b, and c as shown, then:

$a^2 = b^2 + c^2 - 2bc\cos A$

$b^2 = a^2 + c^2 - 2ac\cos B$

$c^2 = \underline{a^2 + b^2 - 2ab\cos C}$



HERON'S AREA FORMULA

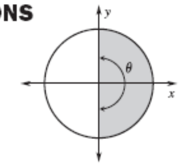
The area of a triangle with sides of length a, b, and c is

Area $= \underline{\sqrt{s(s-a)(s-b)(s-c)}}$

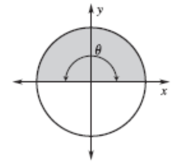
where $s = \underline{\frac{1}{2}(a + b + c)}$. The variable s is called the *semiperimeter*, or half-perimeter, of the triangle.

INVERSE TRIGONOMETRIC FUNCTIONS

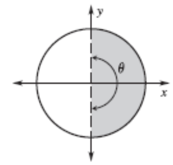
- If $-1 \le a \le 1$, then the $\underline{\text{inverse sine}}$ of a is an angle θ, written $\theta = \sin^{-1} a$, where $\sin\theta = a$ and $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$ (or $-90° \le \theta \le 90°$).



- If $-1 \le a \le 1$, then the $\underline{\text{inverse cosine}}$ of a is an angle θ, written $\theta = \cos^{-1} a$, where $\cos\theta = a$ and $0 \le \theta \le \pi$ (or $0° \le \theta \le 180°$).

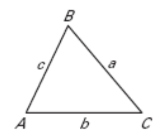


- If a is any real number, then the $\underline{\text{inverse tangent}}$ of a is an angle θ, written $\theta = \tan^{-1} a$, where $\tan\theta = a$ and $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$ (or $-90° < \theta < 90°$).



LAW OF SINES

The law of sines can be written in either of the following forms for $\triangle ABC$ with sides of lengths a, b, and c.

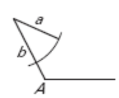


$\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$ $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

POSSIBLE TRIANGLES IN THE SSA CASE

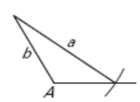
Consider a triangle in which you are given a, b, and A. By fixing side b and angle A, you can sketch the possible positions of side a to figure out how many triangles can be formed. In the diagrams below, note that $h = b \sin A$.

***A* is obtuse.**



$a \le b$

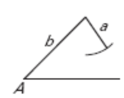
No triangle



$a > b$

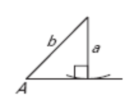
One triangle

***A* is acute.**



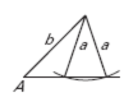
$h > a$

No triangle



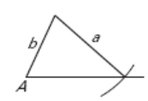
$h = a$

One triangle



$h < a < b$

Two triangles

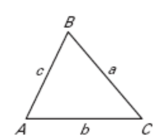


$a > b$

One triangle

AREA OF A TRIANGLE

The area of any triangle is given by one half the product of the lengths of two sides times the sine of their included angle. For $\triangle ABC$ shown, there are three ways to calculate the area:



Area $= \underline{\frac{1}{2}bc\sin A}$ Area $= \underline{\frac{1}{2}ac\sin B}$

Area $= \underline{\frac{1}{2}ab\sin C}$