VOCABULARY

Sequence A function whose domain is a set of consecutive integers

Terms The values in the range of a sequence

Series The expression that results when the terms of a sequence are added together

Summation notation Notation for a series that represents the sum of the terms

Sigma notation Another name for summation notation, which uses the uppercase Greek letter, sigma, written Σ

SEQUENCES

A sequence is a function whose domain is a set of consecutive integers. If a domain is not specified, it is understood that the domain starts with 1. The values in the range are called the terms of the sequence.

Domain:	1	2	3	4 . . . n		**The relative position of each term**
	↓	↓	↓	↓	↓	
Range:	a_1	a_2	a_3	a_4 · · ·	a_n	**Terms of the sequence**

A finite sequence has a limited number of terms.
An infinite sequence continues without stopping.

Finite sequence: 2, 4, 6, 8

Infinite Sequence: 2, 4, 6, 8, · · ·

A sequence can be specified by an equation, or rule. For example, both sequences above can be described by the rule $a_n = 2n$ or $f(n) = 2n$.

SERIES AND SUMMATION NOTATION

When the terms of a sequence are added together, the resulting expression is a series. A series can be finite or infinite.

Finite series: $2 + 4 + 6 + 8$

Infinite series: $2 + 4 + 6 + 8 \cdots$

You can use summation notation to write a series.

$$2 + 4 + 6 + 8 = \sum_{i=1}^{4} 2i$$

$$2 + 4 + 6 + 8 + \cdots = \sum_{i=1}^{\infty} 2i$$

For both series, the index of summation is i and the lower limit of summation is 1. The upper limit of summation is 4 for the finite series and ∞ (infinity) for the infinite series. Summation notation is also called sigma notation because it uses the uppercase Greek letter sigma, written Σ.

FORMULAS FOR SPECIAL SERIES

Sum of n terms of 1	**Sum of first n positive integers**	**Sum of squares of first n positive integers**
$\sum_{i=1}^{n} 1 = n$	$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$	$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

VOCABULARY

Arithmetic sequence A sequence in which the difference between consecutive terms is constant

Common difference The constant difference between terms of an arithmetic sequence, denoted by d

Arithmetic series The expression formed by adding the terms of an arithmetic sequence, denoted by S_n

RULE FOR AN ARITHMETIC SEQUENCE

The nth term of an arithmetic sequence with first term a_1 and common difference d is given by:

$a_n = a_1 + (n - 1)d$

THE SUM OF A FINITE ARITHMETIC SERIES

The sum of the first n terms of an arithmetic series is:

$$S_n = n\left(\frac{a_1 + a_n}{2}\right)$$

In words, S_n is the mean of the first and nth terms, multiplied by the number of terms.

VOCABULARY

Geometric sequence A sequence in which the ratio of any term to the previous term is constant

Common ratio The constant ratio between consecutive terms of a geometric sequence, denoted by r

Geometric series The expression formed by adding the terms of a geometric sequence

RULE FOR A GEOMETRIC SEQUENCE

The nth term of a geometric sequence with first term a_1 and common ratio r is given by: $a_n = a_1 r^{n-1}$

THE SUM OF A FINITE GEOMETRIC SERIES

The sum of the first n terms of a geometric series with common ratio $r \neq 1$ is:

$$S_n = a_1\left(\frac{1 - r^n}{1 - r}\right)$$

VOCABULARY

Partial sum The sum S_n of the first n terms of an infinite series

THE SUM OF AN INFINITE GEOMETRIC SERIES

The sum of an infinite geometric series with first term a_1 and common ratio r is given by

$$S = \frac{a_1}{1 - r}$$

provided $|r| < 1$. If $|r| \geq 1$, the series has no sum.

VOCABULARY

Explicit rule A rule for a sequence that gives a_n as a function of the term's position number n

Recursive rule A rule for a sequence that gives the beginning term or terms of a sequence and then a recursive equation that tells how a_n is related to one or more preceding terms

Iteration The repeated composition of a function f with itself

RECURSIVE EQUATIONS FOR ARITHMETIC AND GEOMETRIC SEQUENCES

Arithmetic Sequence $a_n = a_{n-1} + d$ where d is the common difference

Geometric Sequence $a_n = r \cdot a_{n-1}$ where r is the common ratio