THE DISTANCE FORMULA

The distance d between (x_1, y_1) and (x_2, y_2) is
$d = \sqrt{(\underline{x_2 - x_1})^2 + (\underline{y_2 - y_1})^2}$.

THE MIDPOINT FORMULA

A line segment's midpoint is equidistant from the segment's endpoints. The midpoint formula describes the midpoint of a line segment joining $A(x_1, y_1)$ and $B(x_2, y_2)$ as follows:

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

In words, each coordinate of M is the mean of the corresponding coordinates of A and B.

VOCABULARY

Focus A fixed point that lies on the axis of symmetry of a parabola

Directrix A line that is perpendicular to the axis of symmetry of a parabola

STANDARD EQUATION OF A PARABOLA WITH VERTEX AT THE ORIGIN

The standard form of the equation of a parabola with vertex at (0, 0) is as follows:

Equation	Focus	Directrix	Axis of Symmetry
$x^2 = 4py$	$(0, p)$	$y = -p$	Vertical ($x = 0$)
$y^2 = 4px$	$(p, 0)$	$x = -p$	Horizontal ($y = 0$)

VOCABULARY

Circle The set of all points (x, y) that are equidistant from a fixed point

Center The fixed point that is equidistant from all the points on a circle

Radius The distance r between the center and any point (x, y) on a circle

STANDARD EQUATION OF A CIRCLE WITH CENTER AT THE ORIGIN

The standard form of the equation of a circle with center at (0, 0) and radius r is as follows:

$x^2 + y^2 = r^2$

VOCABULARY

Ellipse The set of all points P such that the sum of the distances between P and two fixed points, called the foci, is a constant

Foci Two fixed points in an ellipse

Vertices The points at which the line through the foci intersect the ellipse

Major axis The line segment that joins the vertices

Center The midpoint of the major axis

Co-vertices The points of intersection of an ellipse and the line perpendicular to the major axis at the center

Minor axis The line segment that joins the co-vertices

STANDARD EQUATION OF AN ELLIPSE WITH CENTER AT THE ORIGIN

Equation	Major Axis	Vertices	Co-Vertices
$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	Horizontal	$(\pm a, 0)$	$(0, \pm b)$
$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$	Vertical	$(0, \pm a)$	$(\pm b, 0)$

The major and minor axes are of lengths $2a$ and $2b$, respectively, where $a > b > 0$. The foci of the ellipse lie on the major axis at a distance of c units from the center, where $c^2 = a^2 - b^2$.

VOCABULARY

Hyperbola The set of all points P such that the difference of the distances between P and two fixed points, called the foci, is a constant

Foci Two fixed points in a hyperbola

Vertices The points of intersection of a hyperbola and the line through the foci

Transverse Axis The line segment that connects the vertices of a hyperbola

Center The midpoint of the transverse axis

STANDARD EQUATION OF A HYPERBOLA WITH CENTER AT THE ORIGIN

Equation	Transverse Axis	Asymptotes	Vertices
$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	Horizontal	$y = \pm \frac{b}{a} x$	$(\pm a, 0)$
$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$	Vertical	$y = \pm \frac{a}{b} x$	$(0, \pm a)$

The foci lie on the transverse axis, c units from the center, where $c^2 = a^2 + b^2$.

VOCABULARY

Conic sections The intersection of a plane and a double-napped cone

General second-degree equation An equation of the form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$

Discriminant The expression $B^2 - 4AC$ for the equation $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, used to identify a conic section

STANDARD FORM OF EQUATIONS OF TRANSLATED CONICS

In the following equations, the point (h, k) is the vertex of the parabola and the center of the other conics.

Circle	$(x - h)^2 + (y - k)^2 = r^2$	
Parabola	$(y - k)^2 = 4p(x - h)$	Horizontal axis
	$(x - h)^2 = 4p(y - k)$	Vertical axis
Ellipse	$\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$	Horizontal axis
	$\frac{(x - h)^2}{b^2} + \frac{(y - k)^2}{a^2} = 1$	Vertical axis
Hyperbola	$\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1$	Horizontal axis
	$\frac{(y - k)^2}{a^2} - \frac{(x - h)^2}{b^2} = 1$	Vertical axis

CLASSIFYING CONICS USING THEIR EQUATIONS

Any conic can be described by a general second-degree equation in x and y:

$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0.$

The expression $B^2 - 4AC$ is the discriminant of the conic equation and can be used to identify it.

Discriminant	Type of Conic
$B^2 - 4AC < 0$, $B = 0$, and $A = C$	Circle
$B^2 - 4AC < 0$ and either $B \neq 0$ or $A \neq C$	Ellipse
$B^2 - 4AC = 0$	Parabola
$B^2 - 4AC > 0$	Hyperbola

If $B = 0$, each axis of the conic is horizontal or vertical.

VOCABULARY

Quadratic system A system that includes one or more equations of conics