VOCABULARY

Inverse variation Two variables x and y show inverse variation if they are related as follows: $y = \frac{a}{x}$, $a \neq 0$.

Constant of variation The nonzero constant a in a variation equation

Joint variation When a quantity varies directly with the product of two or more other quantities

INVERSE VARIATION

Two variables x and y show inverse variation if they are related as follows: $y = \frac{a}{x}$, $a \neq 0$

The constant a is the constant of variation, and y is said to vary inversely with x.

VOCABULARY

Rational function A function of the form $f(x) = \frac{p(x)}{q(x)}$ where $p(x)$ and $q(x)$ are polynomials and $q(x) \neq 0$

PARENT FUNCTION FOR SIMPLE RATIONAL FUNCTIONS

- The graph of the parent function $f(x) = \frac{1}{x}$ is a hyperbola, which consists of two symmetrical parts called branches.
- The domain and range are all nonzero real numbers.
- The asymptotes are $x =$ 0, and $y =$ 0.
- Any function of the form $g(x) = \frac{a}{x}$ ($a \neq 0$) has the same asymptotes, domain, and range as the function $f(x) = \frac{1}{x}$.

GRAPHING TRANSLATIONS OF SIMPLE RATIONAL FUNCTIONS

To graph a rational function of the form $y = \frac{a}{x - h} + k$, follow these steps:

Step 1 Draw the asymptotes $x =$ h and $y =$ k.

Step 2 Plot points to the left and to the right of the vertical asymptote.

Step 3 Draw the two branches of the hyperbola so that they pass through the plotted points and approach the asymptotes.

GRAPHS OF RATIONAL FUNCTIONS

Let $p(x)$ and $q(x)$ be polynomials with no common factors other than ± 1.

$$f(x) = \frac{p(x)}{q(x)} = \frac{a_m x^m + a_{m-1}x^{m-1} + \cdots + a_1 x + a_0}{b_n x^n + b_{n-1}x^{n-1} + \cdots + b_1 x + b_0}$$

1. The x-intercepts of the graph of f are the real zeros of $p(x)$.
2. The graph of f has a vertical asymptote at each real zero of $q(x)$.
3. The graph of f has at most one horizontal asymptote, determined by the degrees m and n of $p(x)$ and $q(x)$.

- If $m < n$, the line $y = 0$ is a horizontal asymptote.
- If $m = n$, the line $y = \frac{a_m}{b_n}$ is a horizontal asymptote.
- If $m > n$, the graph has no horizontal asymptote. The end behavior is the same as $y = \frac{a_m}{b_n}x^{m-n}$.

VOCABULARY

Simplified form of a rational expression A rational expression in which its numerator and denominator have no common factors (other than ± 1)

SIMPLIFYING RATIONAL EXPRESSIONS

Let a, b, and c be nonzero real numbers or variable expressions. Then the following property applies.

$\frac{ac}{bc} = \frac{a}{b}$ Divide out common factor c.

MULTIPLYING RATIONAL EXPRESSIONS

Let a, b, c, and d be nonzero real numbers or variable expressions. The rule for multiplying rational expressions is the same as the rule for multiplying numerical fractions: multiply numerators, multiply denominators, and write the new fraction in simplified form.

$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$ ← Simplify $\frac{ac}{bd}$ if possible.

DIVIDING RATIONAL EXPRESSIONS

To divide one rational expression by another, multiply the first expression by the reciprocal of the second expression.

$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$ ← Simplify $\frac{ad}{bc}$ if possible.

VOCABULARY

Complex fraction A fraction that contains a fraction in its numerator or denominator

ADD (SUBTRACT) WITH LIKE DENOMINATORS

To add (or subtract) rational expressions with like denominators, simply add (or subtract) their numerators. Then place the result over the common denominator. Let a, b, and c be polynomials with $c \neq 0$.

Addition $\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$ **Subtraction** $\frac{a}{c} - \frac{b}{c} = \frac{a-b}{c}$

ADD (SUBTRACT) WITH UNLIKE DENOMINATORS

To add (or subtract) two rational expressions with unlike denominators, find the least common denominator (LCD), which is the least common multiple (LCM) of the denominators.

Rewrite each rational expression using the LCD, then add (or subtract) using the procedure for like denominators. Let a, b, c, and d be polynomials with $c \neq 0$ and $d \neq 0$.

Addition $\frac{a}{c} + \frac{b}{d} = \frac{ad}{cd} + \frac{bc}{cd} = \frac{ad+bc}{cd}$

Subtraction $\frac{a}{c} - \frac{b}{d} = \frac{ad}{cd} - \frac{bc}{cd} = \frac{ad-bc}{cd}$

SIMPLIFYING COMPLEX FRACTIONS

A complex fraction is a fraction that contains a fraction in its numerator or denominator.

Method 1: If necessary, simplify the numerator and denominator by writing each as a single fraction. Then divide the numerator by the denominator.

Method 2: Multiply the numerator and the denominator by the least common denominator (LCD) of every fraction in the numerator and denominator. Then simplify.

VOCABULARY

Cross multiplying A method of solving a simple rational equation for which each side of the equation is a single rational expression. Equal products are formed by multiplying the numerator of each expression by the denominator of the other.