VOCABULARY

Exponential function A function of the form $y = ab^x$ where $a \neq 0$ and the base b is a positive number other than one

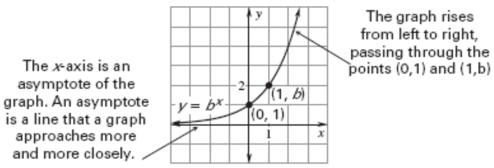
Exponential growth function A function of the form $y = ab^x$ where $a > 0$ and $b > 1$

Growth factor In a function of the form $y = ab^x$, b is the growth factor.

Asymptote A line that a graph approaches more and more closely

PARENT FUNCTION FOR EXPONENTIAL GROWTH FUNCTIONS

The function $y = b^x$, where b $>$ 1, is the parent function for the family of exponential growth functions with base b. The general shape of the graph of $y = b^x$ is shown below.



The domain of $y = b^x$ is all real numbers. The range is $y > 0$.

COMPOUND INTEREST

Consider an initial principal P deposited in an account that pays interest at an annual rate r (expressed as a decimal), compounded n times per year. The amount A in the account after t years is given by this equation:

$A = P\left(1 + \frac{r}{n}\right)^{nt}$

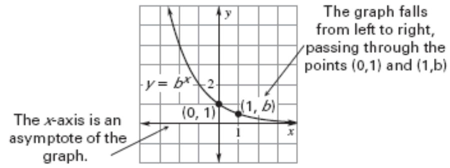
VOCABULARY

Exponential decay function A function of the form $y = ab^x$ where $a > 0$ and $0 < b < 1$

Decay factor In a function of the form $y = ab^x$, the base b is the decay factor.

PARENT FUNCTION FOR EXPONENTIAL DECAY FUNCTIONS

The function $y = b^x$, where $0 < b < 1$, is the parent function for the family of exponential decay functions with base b. The general shape of the graph of $y = b^x$ is shown below.



The domain of $y = b^x$ is all real numbers. The range is $y > 0$.

VOCABULARY

Logarithm of y with base b A logarithm denoted by $\log_b y$ and defined as $\log_b y = x$ if and only if $b^x = y$, given that b and y are positive numbers with $b \neq 1$.

Common logarithm A logarithm with base 10

Natural logarithm A logarithm with base e

THE NATURAL BASE e

The natural base e is irrational. It is defined as follows:

As n approaches $+\infty$, $\left(1 + \frac{1}{n}\right)^n$ approaches $e \approx$ 2.718281828.

NATURAL BASE FUNCTIONS

A function of the form $y = ae^{rx}$ is called a natural base exponential function.

- If $a > 0$ and $r > 0$, the function is an exponential growth function.
- If $a > 0$ and $r < 0$, the function is an exponential decay function.

CONTINUOUSLY COMPOUNDED INTEREST

When interest is compounded continuously, the amount A in an account after t years is given by the formula $A = Pe^{rt}$ where P is the principal and r is the annual interest rate expressed as a decimal.

DEFINITION OF LOGARITHM WITH BASE b

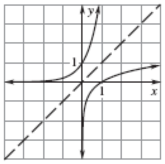
Let b and y be positive numbers with $b \neq 1$. The logarithm of y with base b is denoted by $\log_b y$ and is defined as follows:

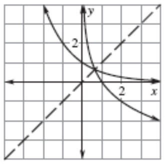
$\log_b y = x$ if and only if $b^x = y$

The expression $\log_b y$ is read as "log base b of y."

PARENT GRAPHS FOR LOGARITHMIC FUNCTIONS

The graph of $y = \log_b x$ is shown below for $b > 1$ and for $0 < b < 1$. Because $y = \log_b x$ and $y = b^x$ are inverse functions, the graph of $y = \log_b x$ is the reflection of the graph of $y = b^x$ in the line $y = x$.





Note that the y-axis is a vertical asymptote of the graph of $y = \log_b x$. The domain of $y = \log_b x$ is $x > 0$, and the range is all real numbers.

PROPERTIES OF LOGARITHMS

Let b, m, and n be positive numbers such that $b \neq 1$.

Product Property	$\log_b mn = \log_b m + \log_b n$
Quotient Property	$\log_b \frac{m}{n} = \log_b m - \log_b n$
Power Property	$\log_b m^n = n \log_b m$

CHANGE OF BASE FORMULA

If a, b, and c are positive numbers with $b \neq 1$ and $c \neq 1$, then

$$\log_c a = \frac{\log_b a}{\log_b c}$$

In particular $\log_c a = \frac{\log a}{\log c}$ and $\log_c a = \frac{\ln a}{\ln c}$.

VOCABULARY

Exponential equation An equation in which variable expressions occur as exponents

Logarithmic equation An equation that involves logarithms of variable expressions

PROPERTY OF EQUALITY FOR EXPONENTIAL EQUATIONS

Algebra If b is a positive number other than 1, then $b^x = b^y$ if and only if $x = y$.

Example If $5^x = 5^4$, then $x = 4$. If $x = 4$, then $5^x = 5^4$.

PROPERTY OF EQUALITY FOR LOGARITHMIC EQUATIONS

Algebra If b, x, and y are positive numbers with $b \neq 1$, then $\log_b x = \log_b y$ if and only if $x = y$.

Example If $\log_3 x = \log_3 8$, then $x = 8$. If $x = 8$, then $\log_3 x = \log_3 8$.