VOCABULARY

*n*th root of *a* For an integer n greater than 1, if $b^n = a$, then b is an nth root of a.

Index of a radical An nth root of a is written as $\sqrt[n]{a}$, where n is the index of the radical.

REAL *n*th ROOTS OF *a*

Let n be an integer ($n > 1$) and let a be a real number.

If n is an even integer:

- $a < 0$ No real nth roots.
- $a = 0$ One real nth root: $\sqrt[n]{0} = 0$
- $a > 0$ Two real nth roots: $\pm\sqrt[n]{a} = \pm a^{1/n}$

If n is an odd integer:

- $a < 0$ One real nth root: $\sqrt[n]{a} = a^{1/n}$
- $a = 0$ One real nth root: $\sqrt[n]{0} = 0$
- $a > 0$ One real nth root: $\sqrt[n]{a} = a^{1/n}$

RATIONAL EXPONENTS

Let a be a real number, and let m and n be positive integers with $n > 1$.

$a^{m/n} = (a^{1/n})^m = (\sqrt[n]{a})^m$

$a^{-m/n} = \frac{1}{a^{m/n}} = \frac{1}{(a^{1/n})^m} = \frac{1}{(\sqrt[n]{a})^m}, a \neq 0$

PROPERTIES OF RATIONAL EXPONENTS

Let a and b be real numbers and let m and n be rational numbers. The following properties have the same names as those in Lesson 5.1, but now apply to rational exponents.

Property	
1. $a^m \cdot a^n = a^{m+n}$	$4^{1/2} \cdot 4^{3/2} = 4^{(1/2 + 3/2)} = 4^2 = 16$
2. $(a^m)^n = a^{mn}$	$(2^{5/2})^2 = 2^{(5/2 \cdot 2)} = 2^5 = 32$
3. $(ab)^m = a^m b^m$	$(16 \cdot 4)^{1/2} = 16^{1/2} \cdot 4^{1/2} = 4 \cdot 2 = 8$
4. $a^{-m} = \frac{1}{a^m}, a \neq 0$	$25^{-1/2} = \frac{1}{25^{1/2}} = \frac{1}{5}$
5. $\frac{a^m}{a^n} = a^{m-n}, a \neq 0$	$\frac{3^{5/2}}{3^{1/2}} = 3^{(5/2 - 1/2)} = 3^2 = 9$
6. $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}, b \neq 0$	$\left(\frac{27}{8}\right)^{1/3} = \frac{27^{1/3}}{8^{1/3}} = \frac{3}{2}$

VOCABULARY

Simplest form of a radical A radical with index n is in simplest form if the radicand has no perfect nth powers as factors and any denominator has been rationalized.

Like radicals Two radical expressions with the same index and radicand.

PROPERTIES OF RADICALS

Product Property of Radicals: $\sqrt[n]{a \cdot b} = \sqrt[n]{a} \cdot \sqrt[n]{b}$

Quotient Property of Radicals: $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}, b \neq 0$

VOCABULARY

Power function A function of the form $y = ax^b$ where a is a real number and b is a rational number

Composition The composition of a function g with a function f is $h(x) = g(f(x))$. The domain of h is the set of all x-values such that x is in the domain of f and $f(x)$ is in the domain of g.

OPERATIONS ON FUNCTIONS

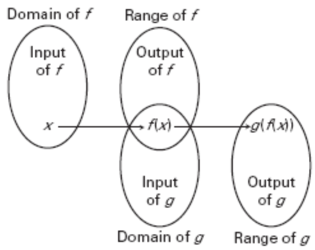
Let f and g be any two functions. A new function h can be defined by performing any of the four basic operations on f and g.

Operation and Definition	Example: $f(x) = 3x$, $g(x) = x + 3$
Addition $h(x) = f(x) + g(x)$	$h(x) = 3x + (x + 3) = 4x + 3$
Subtraction $h(x) = f(x) - g(x)$	$h(x) = 3x - (x + 3) = 2x - 3$
Multiplication $h(x) = f(x) \cdot g(x)$	$h(x) = 3x(x + 3) = 3x^2 + 9x$
Division $h(x) = \frac{f(x)}{g(x)}$	$h(x) = \frac{3x}{x + 3}$

The domain of h consists of the x-values that are in the domains of both f and g. Additionally, the domain of a quotient does not include x-values for which $g(x) = 0$.

COMPOSITION OF FUNCTIONS

The composition of a function g with a function f is $h(x) =$ $g(f(x))$. The domain of h is the set of all x-values such that x is in the domain of f and $f(x)$ is in the domain of g.



VOCABULARY

Inverse relation A relation that interchanges the input and output values of the original relation

Inverse function The original relation and its inverse relation whenever both relations are functions

INVERSE FUNCTIONS

Functions f and g are inverses of each other provided:

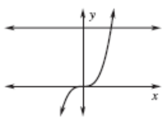
$f(g(x)) =$ x and $g(f(x)) =$ x

The function g is denoted by f^{-1}, read as "f inverse."

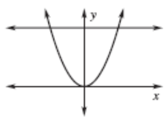
HORIZONTAL LINE TEST

The inverse of a function f is also a function if and only if no horizontal line intersects the graph of f more than once.

Function



Not a function



VOCABULARY

Radical function A function containing a radical such as $y = \sqrt{x}$

PARENT FUNCTIONS FOR SQUARE ROOT AND CUBE ROOT FUNCTIONS

- The parent function for the family of square root functions is $f(x) = \sqrt{x}$. The domain is x ≥ 0, and the range is y ≥ 0.
- The parent function for the family of cube root functions is $g(x) = \sqrt[3]{x}$. The domain and range are all real numbers.

GRAPHS OF RADICAL FUNCTIONS

To graph $y = a\sqrt{x - h} + k$ or $y = a\sqrt[3]{x - h} + k$, follow these steps:

Step 1 Sketch the graph of $y = a\sqrt{x}$ or $y = a\sqrt[3]{x}$.

Step 2 **Translate** the graph h units horizontally and k units vertically.

VOCABULARY

Radical equation An equation with a radical that has variables in the radicand

SOLVING RADICAL EQUATIONS

To solve a radical equation, follow these steps:

Step 1 Isolate the radical on one side of the equation, if necessary.

Step 2 **Raise** each side of the equation to the same power to eliminate the radical and obtain a linear, quadratic, or other polynomial equation.

Step 3 Solve the polynomial equation using techniques you learned in previous chapters. Check your solution.