VOCABULARY

Scientific notation A number is expressed in scientific notation if it is in the form $c \times 10^n$ where $1 \le c < 10$ and n is an integer.

PROPERTIES OF EXPONENTS

Let a and b be real numbers and let m and n be integers.

Product of Powers Property	$a^m \cdot a^n = a^{m+n}$
Power of a Power Property	$(a^m)^n = a^{mn}$
Power of a Product Property	$(ab)^m = a^m b^m$
Negative Exponent Property	$a^{-m} = \frac{1}{a^m}$, $a \neq 0$
Zero Exponent Property	$a^0 = 1$, $a \neq 0$
Quotient of Powers Property	$\frac{a^m}{a^n} = a^{m-n}$, $a \neq 0$
Power of a Quotient Property	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$, $b \neq 0$

VOCABULARY

Polynomial A monomial or a sum of monomials

Polynomial function A function of the form $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ where $a_n \neq 0$, the exponents are all whole numbers, and the coefficients are all real numbers

Synthetic substitution An alternate method to evaluate a polynomial function using fewer operations than direct substitution

End behavior The behavior of a polynomial function's graph as x approaches positive infinity or negative infinity

END BEHAVIOR OF POLYNOMIAL FUNCTIONS

For the graph of
$f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$:

- If $a_n > 0$ and n is odd, then $f(x) \to -\infty$ as $x \to -\infty$ and $f(x) \to +\infty$ as $x \to +\infty$.
- If $a_n < 0$ and n is odd, then $f(x) \to +\infty$ as $x \to -\infty$ and $f(x) \to -\infty$ as $x \to +\infty$.
- If $a_n > 0$ and n is even, then $f(x) \to +\infty$ as $x \to -\infty$ and $f(x) \to +\infty$ as $x \to +\infty$.
- If $a_n < 0$ and n is even, then $f(x) \to -\infty$ as $x \to -\infty$ and $f(x) \to -\infty$ as $x \to +\infty$.

SPECIAL PRODUCT PATTERNS

Sum and Difference	Example
$(a+b)(b-a) = a^2 - b^2$	$(x+2)(x-2) = x^2 - 4$
Square of a Binomial	
$(a+b)^2 = a^2 + 2ab + b^2$	$(y+4)^2 = y^2 + 8y + 16$
$(a-b)^2 = a^2 - 2ab + b^2$	$(3p^2 - 2)^2 = 9p^4 + 12p + 4$
Cube of a Binomial	
$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$	$(x+1)^3 = x^3 + 3x^2 + 3x + 1$
$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$	$(r-3)^3 = r^3 - 9x^2 + 27x - 27$

VOCABULARY

Prime polynomial A polynomial with two or more terms that cannot be written as a product of polynomials of lesser degree using only integer coefficients and constants and the only common factors of its terms are −1 and 1

Factored completely A polynomial is factored completely if it is written as a monomial or the product of a monomial and one or more prime polynomials.

Factor by grouping A method used to factor some polynomials with pairs of terms that have a common monomial factor

Quadratic form An expression of the form $au^2 + bu + c$, where u is any expression in x

FACTORING POLYNOMIALS

Definition A polynomial with two or more terms is a prime polynomial if it cannot be written as a product of polynomials of lesser degree using only integer coefficients and constants and if the only common factors of its terms are −1 and 1.

Example $16x^2 - 4x + 8$ is not a prime polynomial because 4 is a common factor of all its terms.

Definition A polynomial is factored completely if it is written as a monomial or the product of a monomial and one or more prime polynomials.

Example $(x+2)(x^2 - 5x + 6)$ is not factored completely because $x^2 - 5x + 6 = (x-2)(x-3)$.

SPECIAL FACTORING PATTERNS

Sum of Two Cubes

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

Example

$x^3 + 8 = (x+2)(x^2 - 2x + 4)$

Difference of Two Cubes

$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

Example

$8x^3 - 1 = (2x-1)(4x^2 + 2x + 1)$

VOCABULARY

Polynomial long division One method used to divide polynomials similar to the way you divide numbers

Synthetic division A method used to divide any polynomial by a divisor of the form $x - k$

REMAINDER THEOREM

If a polynomial $f(x)$ is divided by $x - k$, then the remainder is $r =$ _$f(k)$_.

FACTOR THEOREM

A polynomial $f(x)$ has a factor $x - k$ if and only if $f(k) =$ _0_.

THE RATIONAL ZERO THEOREM

If $f(x) = a_n x^n + \cdots + a_1 x + a_0$ has _integer_ coefficients, then every rational zero of f has the following form:

$$\frac{p}{q} = \frac{\text{factor of constant term } \boxed{a_0}}{\text{factor of leading coefficient } \boxed{a_n}}$$

VOCABULARY

Repeated Solution For the equation $f(x) = 0$, k is a repeated solution if and only if the factor $(x - k)$ has a degree greater than 1 when f is factored completely.

THE FUNDAMENTAL THEOREM OF ALGEBRA

Theorem: If $f(x)$ is a polynomial of degree n where n _>_ 0, then the equation $f(x) = 0$ has at least _one_ solution in the set of complex numbers.

Corollary: If $f(x)$ is a polynomial of degree n, then the equation $f(x) = 0$ has exactly _n_ solutions provided each solution repeated twice is counted as _2_ solutions, each solution repeated three times is counted as _3_ solutions and so on.

COMPLEX CONJUGATES THEOREM

If f is a polynomial function with _real_ coefficients, and _$a + bi$_ is an imaginary zero of f, then _$a - bi$_ is also a zero of f.

IRRATIONAL CONJUGATES THEOREM

Suppose f is a polynomial function with _rational_ coefficients, and a and b are rational numbers such that $\sqrt{b}$ is irrational. If _$a + \sqrt{b}$_ is a zero of f, then _$a - \sqrt{b}$_ is also a zero of f.

VOCABULARY

Finite differences The differences of consecutive y-values when the x-values in a data set are equally spaced

DESCARTES' RULE OF SIGNS

Let $f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$ be a polynomial function with real coefficients.

- The number of _positive_ real zeros of f is equal to the number of changes in sign of the coefficients of _$f(x)$_ or is less than this by an _even_ number.
- The number of _negative_ real zeros of f is equal to the number of changes in sign of the coefficients of _$f(-x)$_ or is less than this by an _even_ number.

VOCABULARY

Local maximum The y-coordinate of a turning point if the point is higher than all nearby points

Local minimum The y-coordinate of a turning point if the point is lower than all nearby points

ZEROS, FACTORS, SOLUTIONS, AND INTERCEPTS

Let $f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$ be a polynomial function. If k is a real number, then the following statements are equivalent.

Zero: _k_ is a zero of the polynomial function f.

Factor: _$x - k$_ is a factor of the polynomial $f(x)$.

Solution: _k_ is a solution of the polynomial equation $f(x) = 0$.

x-intercept: _k_ is an x-intercept of the graph of the polynomial function f. The graph of f contains (_k_, 0).

TURNING POINTS OF POLYNOMIAL FUNCTIONS

The graph of every polynomial function of degree n has at most _$n - 1$_ turning points. Moreover, if a polynomial function has n distinct real zeros, then its graph has exactly _$n - 1$_ turning points.

PROPERTIES OF FINITE DIFFERENCES

1. If a polynomial function $f(x)$ has degree n, then the nth-order differences of function values for equally-spaced x-values are _nonzero and constant_.
2. Conversely, if the nth-order differences of equally spaced data are _nonzero and constant_, then the data can be represented by a polynomial function of degree n.