VOCABULARY

Quadratic function A function that can be written in the standard form $y = ax^2 + bx + c$ where $a \neq 0$

Parabola The U-shaped graph of a quadratic function

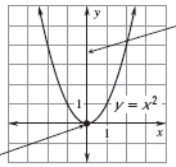
Vertex The lowest or highest point on a parabola

Axis of symmetry The vertical line that divides the parabola into mirror images and passes through the vertex

Minimum and maximum value For $y = ax^2 + bx + c$, the vertex's y-coordinate is the minimum value of the function if $a > 0$ and its maximum value if $a < 0$.

PARENT FUNCTION FOR QUADRATIC FUNCTIONS

The parent function for the family of all quadratic functions is $f(x) =$ x^2. The graph is shown below.



The axis of symmetry divides the parabola into mirror images and passes through the vertex.

The lowest or highest point on a parabola is the vertex. The vertex for $f(x) = x^2$ is (0, 0).

For $f(x) = ax^2$, and for any quadratic function $g(x) = ax^2 + bx + c$ where $b = 0$, the vertex lies on the y-axis and the axis of symmetry is $x =$ 0.

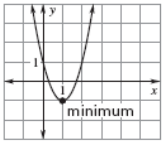
PROPERTIES OF THE GRAPH OF $y = ax^2 + bx + c$

Characteristics of the graph of $y = ax^2 + bx + c$:

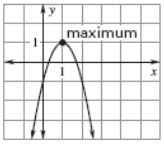
- The graph opens up if $a >$ 0 and opens down if $a <$ 0.
- The graph is narrower than the graph of $y = x^2$ if $|a| >$ 1 and wider if $|a| <$ 1.
- The axis of symmetry is $x = -\frac{b}{2a}$ and the vertex has x-coordinate $-\frac{b}{2a}$.
- The y-intercept is c. So, the point (0, c) is on the parabola.

MINIMUM AND MAXIMUM VALUES

Words For $y = ax^2 + bx + c$, the vertex's y-coordinate is the minimum value of the function if $a >$ 0 and the maximum value if $a <$ 0.



a is positive



a is negative.

GRAPH OF VERTEX FORM $y = a(x - h)^2 + k$

The graph of $y = a(x - h)^2 + k$ is the parabola $y = ax^2$ translated horizontally h units and vertically k units.

- The vertex is (h, k).
- The axis of symmetry is $x =$ h.
- The graph opens up if $a >$ 0 and down if $a <$ 0.

GRAPH OF INTERCEPT FORM $y = a(x - p)(x - q)$

Characteristics of the graph $y = a(x - p)(x - q)$:

- The x-intercepts are p and q.
- The axis of symmetry is halfway between (p, 0) and (q, 0). It has equation $x = \frac{p + q}{2}$.
- The graph opens up if $a >$ 0 and opens down if $a <$ 0.

FOIL METHOD

Words To multiply two expressions that each contain two terms, add the products of the First terms, the Outer terms, the Inner terms, and the Last terms.

Example F O I L

$(x + 4)(x + 7) = x^2 + 7x + 4x + 28 = x^2 + 11x + 28$

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Monomial An expression that is either a number, a variable, or the product of a number and one or more variables

Binomial The sum of two monomials

Trinomial The sum of three monomials

Quadratic equation An equation in one variable that can be written in the form $ax^2 + bx + c = 0$ where $a \neq 0$

Root of an equation A solution of a quadratic function

Zero of a function The numbers p and q of a function in intercept form are also called the zeros of the function.

SPECIAL FACTORING PATTERNS

Pattern Name	
Difference of Two Squares	$a^2 - b^2 = (a + b)(a - b)$ $x^2 - 4 = (x + 2)(x - 2)$
Perfect Square Trinomial	$a^2 + 2ab + b^2 = (a + b)^2$ $x^2 + 6x + 9 = (x + 3)^2$
Perfect Square Trinomial	$a^2 - 2ab + b^2 = (a - b)^2$ $x^2 - 4x + 4 = (x - 2)^2$

ZERO PRODUCT PROPERTY

Words If the product of two expressions is zero, then one or both of the expressions equals zero.

Algebra If A and B are expressions and $AB = 0$, then $A = 0$ or $B = 0$.

Example If $(x + 5)(x + 2) = 0$, then $x + 5 = 0$ or $x + 2 = 0$. That is, $x = -5$ or $x = -2$.

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Square root A number r is a square root of a number s if $r^2 = s$.

Radical An expression of the form $\sqrt{s}$ where s is a number or expression

Radicand The number s beneath the radical sign

Rationalizing the denominator The process of eliminating a radical from the denominator of a fraction

Conjugates The expressions $a + \sqrt{b}$ and $a - \sqrt{b}$, used to rationalize the denominator, whose product is always a rational number

PROPERTIES OF SQUARE ROOTS ($a > 0$, $b > 0$)

		Example
Product Property	$\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$	$\sqrt{18} = \sqrt{9} \cdot \sqrt{2} = 3\sqrt{2}$
Quotient Property	$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$	$\sqrt{\frac{2}{25}} = \frac{\sqrt{2}}{\sqrt{25}} = \frac{\sqrt{2}}{5}$

THE SQUARE ROOT OF A NEGATIVE NUMBER

Property	Example
1. If r is a positive real number, then $\sqrt{-r} = i\sqrt{r}$.	$\sqrt{-3} = i\sqrt{3}$
2. By Property (1), it follows that $(i\sqrt{r})^2 = -r$.	$(i\sqrt{3})^2 = i^2 \cdot 3 = -3$

ABSOLUTE VALUE OF A COMPLEX NUMBER

The absolute value of a complex number $z = a + bi$, denoted $|z|$, is a nonnegative real number defined as $|z| = \sqrt{a^2 + b^2}$. This is the distance of z from the origin in the complex plane.

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Imaginary unit i The imaginary unit i is defined as $i = \sqrt{-1}$.

Complex number A number $a + bi$ where a and b are real numbers. The number a is the real part of the complex number, and the number bi is the imaginary part.

Imaginary number A complex number $a + bi$ where $b \neq 0$

Complex conjugates Two complex numbers of the form $a + bi$ and $a - bi$

Complex plane A coordinate plane where each point (a, b) represents a complex number $a + bi$. The horizontal axis is the real axis and the vertical axis is the imaginary axis.

Absolute value of a complex number The absolute value of a complex number $z = a + bi$, denoted $|z|$, is a nonnegative real number defined as $|z| = \sqrt{a^2 + b^2}$.

SUMS AND DIFFERENCES OF COMPLEX NUMBERS

To add (or subtract) two complex numbers, add (or subtract) their real parts and their imaginary parts separately.

Sum of complex numbers:
$(a + bi) + (c + di) = (a + c) + (b + d)i$

Difference of complex numbers:
$(a + bi) - (c + di) = (a - c) + (b - d)i$

COMPLETING THE SQUARE

Words To complete the square for the expression $x^2 + bx$, add $\left(\frac{b}{2}\right)^2$.

Algebra $x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)\left(x + \frac{b}{2}\right)$

$= \left(x + \frac{b}{2}\right)^2$

THE QUADRATIC FORMULA

Let a, b, and c be real numbers such that $a \neq 0$. The solutions of the quadratic equation $ax^2 + bx + c$ are:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

USING THE DISCRIMINANT OF $ax^2 + bx + c = 0$

When $b^2 - 4ac > 0$, the equation has two real solutions. The graph has two x-intercepts.

When $b^2 - 4ac = 0$, the equation has one real solution. The graph has one x-intercept.

When $b^2 - 4ac < 0$, the equation has two imaginary solutions. The graph has no x-intercepts.

GRAPHING A QUADRATIC INEQUALITY IN TWO VARIABLES

To graph a quadratic inequality, follow these steps:

Step 1 **Graph** the parabola with equation $y = ax^2 + bx + c$.

Make the parabola dashed for inequalities with $<$ or $>$ and solid for inequalities with $\leq$ or $\geq$.

Step 2 **Test** a point (x, y) inside the parabola to determine whether the point is a solution of the inequality.

Step 3 **Shade** the region inside the parabola if the point from Step 2 is a solution. Shade the region outside the parabola if it is not a solution.