VOCABULARY

System of two linear equations Two equations, with the variables x and y, that can be written as:

$Ax + By = C$ Equation 1
$Dx + Ey = F$ Equation 2

Solution of a system An ordered pair (x, y) that satisfies each equation

Consistent A system that has at least one solution

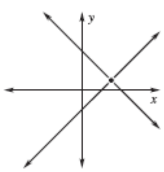
Inconsistent A system that has no solution

Independent A consistent system that has exactly one solution

Dependent A consistent system that has infinitely many solutions

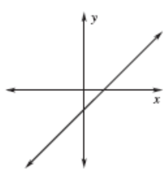
NUMBER OF SOLUTIONS OF A LINEAR SYSTEM

Exactly one solution



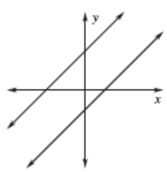
Lines intersect at one point consistent and independent

Infinitely many solutions



Lines coincide; consistent and dependent

No solutions



Lines are parallel; inconsistent

VOCABULARY

Substitution method Substitute an expression into one of the equations to solve for the variable

Elimination method Eliminate one of the variables by adding equations

THE SUBSTITUTION METHOD

Step 1 Solve one of the equations for one of its variables.

Step 2 Substitute the expression from Step 1 into the other equation and solve for the other variable.

Step 3 Substitute the value from Step 2 into the revised equation from Step 1 and solve.

THE ELIMINATION METHOD

Step 1 Multiply one or both of the equations by a constant to obtain coefficients that differ only in sign for one of its variables.

Step 2 Add the revised equations from Step 1. Combining like terms will eliminate one of the variables. Solve for the remaining variable.

Step 3 Substitute the value obtained in Step 2 into either of the original equations and solve for the other variable.

VOCABULARY

System of linear inequalities A system of two or more linear inequalities in two variables

Solution of a system of linear inequalities An ordered pair that is a solution of each inequality in the system

Graph of a system of linear inequalities The graph of all solutions of the system

GRAPHING A SYSTEM OF LINEAR INEQUALITIES

To graph a system of linear inequalities, follow these steps:

Step 1 Graph each inequality in the system. You may want to use colored pencils to distinguish the different half-planes.

Step 2 Identify the region that is common to all the graphs of the inequalities. This region is the graph of the system. If you used colored pencils, the graph of the system is the region that has been shaded with every color.

VOCABULARY

Linear equation in three variables An equation of the form $ax + by + cz = d$, where a, b, and c are not all zero

System of three linear equations A system made up of three linear equations in three variables

Solution of a system of three linear equations The solution is the values of the three variables that make each equation true.

Ordered triple A coordinate in three variables (x, y, z)

THE ELIMINATION METHOD FOR A THREE-VARIABLE SYSTEM

Step 1 Rewrite the linear system in three variables as a linear system in two variables by using the elimination method.

Step 2 Solve the new linear system for both of its variables.

Step 3 Substitute the values found in Step 2 into one of the original equations and solve for the remaining variable.

If you obtain a false equation, such as $0 = 1$, in any of the steps, then the system has no solution.

If you do not obtain a false equation, but obtain an identity such as $0 = 0$, then the system has infinitely many solutions.

VOCABULARY

Matrix A rectangular arrangement of numbers in rows and columns

Dimensions The dimensions of a matrix with m rows and n columns are $m \times n$.

Elements The numbers in a matrix

Equal matrices Matrices that have the same dimensions and equal elements in corresponding positions

Scalar A real number by which you multiply a matrix

Scalar multiplication The process of multiplying each element in a matrix by a scalar

ADDING AND SUBTRACTING MATRICES

To add or subtract two matrices, simply add or subtract corresponding elements. You can add or subtract matrices only if they have the same dimensions.

Adding Matrices $\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} a+e & b+f \\ c+g & d+h \end{bmatrix}$

Subtracting Matrices $\begin{bmatrix} a & b \\ c & d \end{bmatrix} - \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} a-e & b-f \\ c-g & d-h \end{bmatrix}$

PROPERTIES OF MATRIX OPERATIONS

Let A, B, and C be matrices with the same dimensions, and let k be a scalar.

Associative Property of Addition $(A + B) + C = A + (B + C)$

Commutative Property of Addition $A + B = B + A$

Distributive Property of Addition $k(A + B) = kA + kB$

Distributive Property of Subtraction $k(A - B) = kA - kB$

MULTIPLYING MATRICES

Words To find the element in the ith row and jth column of the product matrix AB, multiply each element in the ith row of A by the corresponding element in the jth column of B, then add the products.

Algebra $\underset{A}{\begin{bmatrix} a & b \\ c & d \end{bmatrix}} \cdot \underset{B}{\begin{bmatrix} e & f \\ g & h \end{bmatrix}} = \underset{AB}{\begin{bmatrix} ae+bg & af+bh \\ ce+dg & cf+dh \end{bmatrix}}$

PROPERTIES OF MATRIX MULTIPLICATION

Let A, B, and C be matrices and let k be a scalar.

Associative Property of Matrix Multiplication $A(BC) = (AB)C$

Left Distributive Property $A(B + C) = AB + AC$

Right Distributive Property $(A + B)C = AC + BC$

Associative Property of Scalar Multiplication $k(AB) = (kA)B = A(kB)$

VOCABULARY

Determinant A real number associated with any square matrix A and denoted by det A or $|A|$

Cramer's Rule A method to solve a system of linear equations using the determinants of matrices

Coefficient matrix The coefficient matrix of the linear system $\begin{matrix} ax + by = e \\ cx + dy = f \end{matrix}$ is $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$.

THE DETERMINANT OF A MATRIX

Determinant of a 2 × 2 matrix

$$\det \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - cb$$

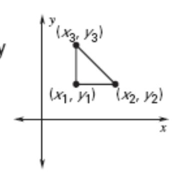
Determinant of a 3 × 3 matrix

$$\det \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} =$$

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}\begin{matrix} a & b \\ d & e \\ g & h \end{matrix} = (aei + bfg + cdh) - (gec + hfa + idb)$$

AREA OF A TRIANGLE

The area of a triangle with vertices (x_1, y_1), (x_2, y_2), and (x_3, y_3) is given by



$$\text{Area} = \pm \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

where the symbol ± indicates that the appropriate sign should be chosen to yield a positive value.

CRAMER'S RULE FOR A 2 × 2 SYSTEM

Let A be the coefficient matrix of this linear system:

$ax + by = e$

$cx + dy = f$

If $\det A \neq$ 0, then the system has exactly one solution.

$$x = \frac{\begin{vmatrix} e & b \\ f & d \end{vmatrix}}{\det A} \quad \text{and} \quad y = \frac{\begin{vmatrix} a & e \\ c & d \end{vmatrix}}{\det A}$$

CRAMER'S RULE FOR A 3 × 3 SYSTEM

Let A be the coefficient matrix of the linear system:

$ax + by + cz = j$

$dx + ey + fz = k$

$gx + hy + iz = \ell$

If $\det A \neq$ 0, then the system has exactly one solution.

$$x = \frac{\begin{vmatrix} j & b & c \\ k & e & f \\ \ell & h & i \end{vmatrix}}{\det A}, \quad y = \frac{\begin{vmatrix} a & j & c \\ d & k & f \\ g & \ell & i \end{vmatrix}}{\det A}, \quad z = \frac{\begin{vmatrix} a & b & j \\ d & e & k \\ g & h & \ell \end{vmatrix}}{\det A}$$

VOCABULARY

Identity matrix An $n \times n$ matrix with 1's on the main diagonal and 0's elsewhere

Inverse matrices Two $n \times n$ matrices A and B are inverses of each other if their product (in both orders) is the $n \times n$ identity matrix.

Matrix of variables The matrix of variables of the linear system $\begin{matrix} ax + by = e \\ cx + dy = f \end{matrix}$ is $\begin{bmatrix} x \\ y \end{bmatrix}$.

Matrix of constants The matrix of constants of the linear system $\begin{matrix} ax + by = e \\ cx + dy = f \end{matrix}$ is $\begin{bmatrix} e \\ f \end{bmatrix}$.

THE INVERSE OF A 2 × 2 MATRIX

The inverse of the matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is

$$A^{-1} = \frac{1}{|A|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{ad - cb}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

provided $ad - cb \neq 0$.

USING AN INVERSE MATRIX TO SOLVE A LINEAR SYSTEM

Step 1 Write the system as a matrix equation $AX = B$. The matrix A is the coefficient matrix, X is the matrix of variables, and B is the matrix of constants.

Step 2 Find the inverse of matrix A.

Step 3 Multiply each side of $AX = B$ by A^{-1} on the left to find the solution $X = A^{-1}B$.