Chapter 14-1 Constructing Parallel Lines
Chapter 14-2 The Meaning of Locus

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. [12]

1. What is the locus of the midpoints of the radii of a circle?
   (1) a line
   (2) a point
   ✓ (3) a circle
   (4) infinitely many circles

2. What is the locus of points equidistant from the sides of an angle?
   (1) a point
   ✓ (2) a line
   (3) two points
   (4) two lines

3. What is the locus of the center of a train wheel that is moving along a straight level track?
   ✓ (1) one line
   (2) two lines
   (3) one circle
   (4) two circles

4. The locus of points in a plane that are a given distance $d$ from a point $P$ is
   ✓ (1) one circle.
   (2) two circles.
   (3) one circle and one point.
   (4) two parallel lines.

5. Point $M$ is on line $\overrightarrow{PQ}$. The locus of the centers of all circles of radius 5 inches which pass through point $M$ is
   ✓ (1) a circle with radius 5 inches and center $M$.
   (2) a line passing through $M$ and perpendicular to $\overrightarrow{PQ}$.
   (3) two lines both perpendicular to $\overrightarrow{PQ}$ and 5 inches on either side of $M$.
   (4) two lines parallel to $\overrightarrow{PQ}$, one 5 inches above $\overrightarrow{PQ}$ and the other 5 inches below $\overrightarrow{PQ}$.

6. The locus of points equidistant from two concentric circles whose radii are 9 inches and 15 inches is
   ✓ (1) one concentric circle of radius 6 inches.
   (2) two concentric circle of radius 6 inches and 12 inches.
   (3) one concentric circle of radius 12 inches.
   (4) one concentric circle of radius 24 inches.
PART II

Answer all questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [8]

7. Construct all lines parallel to \(\overrightarrow{PQ}\) at a distance \(RS\) from \(\overrightarrow{PQ}\).

8. a. Construct the locus of points equidistant from the vertices of \(\triangle ABC\) given below.

   Answer: The point of intersection of the perpendicular bisectors of the sides of \(\triangle ABC\); that is, the circumcenter.

   b. Describe the locus constructed in a.
1. The locus of the center of a dime as it rolls down a ramp is
   (1) a point. (3) two lines.
   ✔ (2) a line. (4) a circle.

2. The path of a swimmer who swims in such a way as to be always equidistant from two buoys is
   (1) a point. (3) two lines.
   ✔ (2) a line. (4) a circle.

3. Triangle ABC is isosceles with base angles \( \angle B \) and \( \angle C \). How many points that lie on the altitude from vertex A to side BC are equidistant from points B and C?
   (1) 0
   (2) 1
   (3) 2
   ✔ (4) infinitely many

4. The number of points on a given line \( m \) that are 4 centimeters from a point \( P \) on that line is
   (1) 0
   (2) 1
   ✔ (3) 2
   (4) infinitely many

5. The locus of the centers of all circles which are tangent to both sides of a given angle is
   (1) a line segment.
   (2) a line segment.
   ✔ (3) a ray.
   (4) two rays.

6. Point P is on line segment \( \overline{AB} \). The locus of the centers of all circles of radius 3 inches which pass through point P is
   (1) a line passing through P and perpendicular to \( \overline{AB} \).
   (2) two lines both perpendicular to \( \overline{AB} \) and 3 inches on either side of P.
   ✔ (3) a circle with radius 3 inches and center P.
   (4) two lines parallel to \( \overline{AB} \), one 3 inches above \( \overline{AB} \) and the other 3 inches below \( \overline{AB} \).
PART II

Answer all questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [8]

7. a. Construct a chord parallel to the given chord of the circle through point S.

b. What is the locus of the midpoints of all chords drawn parallel to the chord given in part a?
   
   **Answer:** A diameter of the circle

8. Describe the locus of points that are:

   a. 8 centimeters from a given line in a plane.
      
      **Answer:** Two lines parallel to the given line and 8 centimeters from the line.

   b. equidistant from two concentric circles whose radii are 6 inches and 10 inches.
      
      **Answer:** The concentric circle with radius of 8 inches.
1. Points $A$ and $B$ lie on line $m$. Line $k$ is parallel to line $m$. How many points are equidistant from $A$ and $B$ and also equidistant from lines $k$ and $m$?

(1) 0

✓ (2) 1

(3) 2

(4) 4

2. To circumscribe a circle about a triangle, you must first construct

(1) two altitudes.

(2) two angle bisectors.

(3) two medians.

✓ (4) the perpendicular bisectors of two sides.

3. Parallel lines $l$ and $m$ are 6 inches apart and $P$ is a point on line $l$. The total number of points in the plane that are equidistant from $l$ and $m$ and also 3 inches from $P$ is

(1) 0

✓ (2) 1

(3) 2

(4) 3

4. Point $P$ is on a line. How many points are at a distance of 4 units from $P$ and also at a distance of 3 units from the given line?

(1) 1

(2) 2

(3) 3

✓ (4) 4

5. How many points in the interior of a given $\angle A$ are equidistant from the sides of $\angle A$ and also 2 centimeters from point $A$?

✓ (1) 1

(2) 2

(3) 3

(4) 4

6. Two straight lines $\overrightarrow{AB}$ and $\overrightarrow{CD}$ intersect at $P$. What is the number of points that are 3 centimeters from $P$ and also equidistant from both $\overrightarrow{AB}$ and $\overrightarrow{CD}$?

(1) 1

(2) 2

(3) 3

✓ (4) 4
PART II

Answer all questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [8]

7. Points $A$ and $B$ are 3 inches apart. Sketch and describe the locus of points:

a. 1 inch from point $A$ and 2 inches from point $B$.
   Answer: A single point on $AB$ one inch below $A$.

b. 2 inches from point $A$ and 2 inches from point $B$.
   Answer: Two points to the left and right of $AB$ and on its perpendicular bisector.

c. 4 inches from point $A$ and 1 inch from point $B$.
   Answer: One point on $AB$ one inch below $B$.

d. 1 inch from point $A$ and 5 inches from point $B$.
   Answer: No such points exist.

8. Given: Point $M$ is the midpoint of $AB$.

a. Describe the locus of points in a plane that are:
   (1) equidistant from points $A$ and $B$.
      Answer: The perpendicular bisector of $AB$

   (2) 4 units from $AB$.
      Answer: The locus is the union of the two segments that are both congruent to and parallel to the altitude and 4 units away from $AB$, and two semicircles of radius 4 centimeters centered at the endpoints of $AB$.

   (3) $d$ units from $M$.
      Answer: A circle of radius $d$ centered at $M$.

b. Describe the value(s) (if any) for $d$ under which there would be:
   (1) no points that satisfy all three conditions in part a.
      Answer: $d < 4$ or $d > 4$

   (2) two points that satisfy all three conditions in part a.
      Answer: $d = 4$

   (3) four points that satisfy all three conditions in part a.
      Answer: No values of $d$ exist.
1. Which equation represents the locus of all points 5 units below the x-axis?

✓ (1) \( y = -5 \)

(2) \( y = 5 \)

(3) \( x = -5 \)

(4) \( x = 5 \)

2. The coordinates of a point on the y-axis equidistant from the two lines, \( y = 6 \), and \( y = -2 \), are

(1) \((-2, 0)\)

(2) \((2, 0)\)

(3) \((0, -2)\)

✓ (4) \((0, 2)\)

3. The equations of the locus of points 5 units from the line whose equation is \( x = 10 \) are

(1) \( x = -5 \) and \( x = -15 \)

(2) \( x = -5 \) and \( x = 15 \)

(3) \( x = 5 \) and \( x = -15 \)

✓ (4) \( x = 5 \) and \( x = 15 \)

4. Which of the following is the equation for the locus of all points at a distance \( \sqrt{4} \) from the point \((-2, 1)\)?

(1) \((x - 2)^2 + (y + 1)^2 = 16\)

(2) \((x - 2)^2 + (y + 1)^2 = 4\)

✓ (3) \((x + 2)^2 + (y - 1)^2 = 4\)

(4) \((x + 2)^2 + (y - 1)^2 = 16\)

5. The equations of the locus of points that are 6 units from the origin are

(1) \( x = 6 \) and \( x = -6 \)

(2) \( y = 6 \) and \( y = -6 \)

(3) \( x^2 + y^2 = 6 \)

✓ (4) \( x^2 + y^2 = 36 \)

6. For which pair of equations is the locus of points exactly one point?

✓ (1) \((x - 1)^2 + (y + 1)^2 = 9 \) and \( x = -2 \)

(2) \((x - 1)^2 + (y + 1)^2 = 9 \) and \( x = 3 \)

(3) \((x - 1)^2 + (y + 1)^2 = 9 \) and \( y = -3 \)

(4) \((x - 1)^2 + (y + 1)^2 = 9 \) and \( y = 1 \)
PART II

Answer all questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [8]

7. Write the equation(s) or coordinates of the locus of points that are:
   a. equidistant from \( x^2 + y^2 = 4 \) and \( x^2 + y^2 = 64 \).

   **Answer:** \( x^2 + y^2 = 25 \)

   **Solution:**
   The radii of the circles are 2 and 8. The locus of points equidistant to these circles is a circle of radius 5.

   b. on the line \( x = 2y \) and on the circle \( x^2 + y^2 = 15 \).

   **Answer:** \((-2\sqrt{3}, -\sqrt{3})\) and \((2\sqrt{3}, \sqrt{3})\)

   **Solution:**
   Substitute \( x = 2y \) into the equation of the circle.

   
   \[
   \begin{align*}
   x^2 + y^2 &= 15 \\
   (2y)^2 + y^2 &= 15 \\
   5y^2 &= 15 \\
   y^2 &= 3 \\
   y &= \pm \sqrt{3}
   \end{align*}
   \]

   Therefore, the points are \((-2\sqrt{3}, -\sqrt{3})\) and \((2\sqrt{3}, \sqrt{3})\).

8. Write the equation(s) or coordinates of the locus of points that are:
   a. 10 units from the origin.

   **Answer:** \( x^2 + y^2 = 100 \)

   b. 6 units from the y-axis.

   **Answer:** \( x = -6 \) and \( x = 6 \)

   c. 10 units from the origin and 6 units from the y-axis.

   **Answer:** \((-6, 8), (-6, -8), (6, 8), (6, -8)\)

   **Solution:**
   Substitute \( x = 6 \) and \( x = -6 \) into the equation of the circle.

   \[
   \begin{align*}
   x^2 + y^2 &= 100 \\
   6^2 + y^2 &= 100 \\
   \quad y &= \sqrt{64} = \pm 8
   \end{align*}
   \]

   Thus, the points are \((-6, 8), (-6, -8), (6, 8), \) and \((6, -8)\).
1. Which of the following is an equation of the locus of points for which the \( x \)-coordinate is 3 less than 5 times the \( y \)-coordinate?

\[ \begin{align*}
(1) & \quad x = 5y - 3 \\
(2) & \quad y = 5x - 3 \\
(3) & \quad x = 5y + 3 \\
(4) & \quad y = 5x + 3
\end{align*} \]

\[ ✔ \quad (1) \]

2. Which of the following is the locus of points equidistant from the points \( A(2, 3) \) and \( B(-4, 3) \)?

\[ \begin{align*}
(1) & \quad x = 1 \\
(2) & \quad x = -1 \\
(3) & \quad y = -1 \\
(4) & \quad y = 1
\end{align*} \]

\[ ✔ \quad (2) \]

3. Which is an equation of the locus of points equidistant from lines \( y = 2x + 3 \) and \( y = 2x - 7 \)?

\[ \begin{align*}
(1) & \quad y = 2x - 5 \\
(2) & \quad y = 2x - 2 \\
(3) & \quad y = -1.5x - 5 \\
(4) & \quad y = -1.5x - 2
\end{align*} \]

\[ ✔ \quad (2) \]

4. Which of the following points is both equidistant from the pair of points \( A(1, 2) \) and \( B(7, 2) \) and 3 units from \( P(4, 1) \)?

\[ \begin{align*}
(1) & \quad (-3, 4) \\
(2) & \quad (4, -2) \\
(3) & \quad (1, 1) \\
(4) & \quad (7, 1)
\end{align*} \]

\[ ✔ \quad (2) \]

5. Which of the following is not the equation of the locus of points equidistant from the \( x \)-axis and the \( y \)-axis and whose coordinates have the same sign?

\[ \begin{align*}
(1) & \quad \frac{y}{x} = 1 \\
(2) & \quad -y = -x \\
(3) & \quad y = x \\
(4) & \quad y = -x
\end{align*} \]

\[ ✔ \quad (4) \]

6. Which of the following is the equation of the locus of points equidistant from the lines \( y = -x - 5 \) and \( y = -x + 1 \)?

\[ \begin{align*}
(1) & \quad y = -x - 2 \\
(2) & \quad y = x - 2 \\
(3) & \quad y = x - 4 \\
(4) & \quad y = -x - 4
\end{align*} \]

\[ ✔ \quad (1) \]
PART II

Answer all questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [8]

7. Write the equation for the locus of points equidistant from $P(-2, 4)$ and $Q(6, 8)$.

**Solution:**

$$y = -2x + 10$$

**Answer:**

The locus of points is the perpendicular bisector of $PQ$.

The midpoint of $PQ$ is $\left(\frac{-2 + 6}{2}, \frac{4 + 8}{2}\right)$ or $(2, 6)$.

The slope of $PQ$ is $\frac{4 - 8}{6}$ or $\frac{-2}{3}$.

Therefore, the locus is the line through $(2, 6)$ with slope $-2$:

$$y - 6 = -2(x - 2)$$
$$y = -2x + 4 + 6$$
$$y = -2x + 10$$

8. The vertices of $\triangle ABC$ are $A(-3, -1)$, $B(3, 7)$, and $C(3, -1)$.

a. Write the equation of the locus of points equidistant from $B$ and $C$.

**Answer:** $y = 3$

b. Write the equation of the line parallel to $BC$ and passing through vertex $A$.

**Answer:** $x = -3$.

c. Find the coordinates for the point of intersection of the locus in part a and the line determined in part b.

**Answer:** $(-3, 3)$

d. Write the equation of the locus of points that are $2\sqrt{13}$ units from vertex $C$.

**Answer:** $(x - 3)^2 + (y + 1)^2 = 52$

e. What is the total number of points that satisfy the loci described in parts a and d?

**Answer:** 2
Chapter 14-6 Points Equidistant from a Point and a Line

PART I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. [12]

1. If \( y = 2x^2 - 3x - 1 \) is graphed, which of the following is the axis of symmetry?
   (1) \( x = \frac{3}{2} \)
   (2) \( x = \frac{3}{4} \)
   (3) \( y = \frac{3}{4} \)
   (4) \( y = -\frac{3}{2} \)
   ✔ (2) \( x = \frac{3}{4} \)

2. Which of the following could be the equation of the parabola shown above?
   (1) \( y = x^2 - 2 \)
   (2) \( y = -x^2 \)
   (3) \( y = -x^2 + 2 \)
   (4) \( y = -x^2 - 2 \)
   ✔ (4) \( y = -x^2 - 2 \)

3. What is the \( y \)-coordinate of the turning point of the graph of \( y = -x^2 + 4x + 1 \)?
   (1) 2
   ✔ (3) 5
   (2) 4
    (4) 13

4. Which of the following is an intersection point of the graphs of the equations \( y = x \) and \( y = x^2 + x - 1 \)?
   (2) (1, 0)
   (1) (0, 0)
   (3) (−1, 0)
   ✔ (4) (−1, −1)

5. The parabola defined by \( y = x^2 + bx + c \) crosses the \( x \)-axis at (1, 0). If the equation of the axis of symmetry is \( x = -2 \), then what is the value of \( c \)?
   ✔ (1) −5
   (3) 4
   (2) −4
   (4) 5

6. What is the turning point of the locus of points equidistant from (2, 1) and the line \( y = 3 \)?
   ✔ (1) (2, 2)
   (2) (2, 3)
   (3) (0, 1.5)
   (4) (0, 3)
7. a. Find the equations or coordinates of the locus of points that satisfy the two equations below:

\[ y = x^2 + 3 \] and \[ y = 3x + 1 \]

**Answer:** (2, 7) and (1, 4)

**Solution:**
Substitute \( y = 3x + 1 \) into the equation of the parabola and solve for \( x \):

\[
\begin{align*}
3x + 1 &= x^2 + 3 \\
0 &= x^2 - 3x + 2 \\
0 &= (x - 2)(x - 1)
\end{align*}
\]

\[
\begin{align*}
0 &= x - 2 & 0 &= x - 1 \\
x &= 2 & x &= 1
\end{align*}
\]

Therefore, the locus of points are (2, 7) and (1, 4).

b. Find the equations or coordinates of the locus of points that satisfy the two equations below:

\[ y = 6 - x \] and \[ y = x^2 - 6x + 6 \]

**Answer:** (0, 6) and (5, 1)

**Solution:**
Substitute \( y = 6 - x \) into the equation of the parabola and solve for \( x \):

\[
\begin{align*}
6 - x &= x^2 - 6x + 6 \\
0 &= x^2 - 5x \\
0 &= x(x - 5)
\end{align*}
\]

\[
\begin{align*}
x &= 0 & x - 5 &= 0 \\
x &= 5 & x &= 5
\end{align*}
\]

Therefore, the locus of points are (0, 6) and (5, 1).
8. Given: The point (0, 3) and the line $y = -3$.
   a. Write an equation of the locus of points that are equidistant from the given point and the given line.
      
      **Answer:** $y = \frac{x^2}{12}$
      
      **Solution:**
      We have that $d = 3$.
      The turning point is $(h, k) = (0, 0)$.
      Therefore, the equation of the parabola is:
      
      $4d(y - k) = (x - h)^2$
      $4(3)(y - 0) = (x - 0)^2$
      $12y = x^2$
      $y = \frac{x^2}{12}$

   b. State the turning point and the axis of symmetry of the equation in **a**.
      
      **Answer:** Turning point: (0, 0), axis of symmetry: $x = 0$

   c. Is the point $(-6, 3)$ is equidistant from the given point and the given line? Justify your answer.
      
      **Answer:** Yes
      
      **Solution:**
      
      $y = \frac{x^2}{12}$
      $3 = \frac{(-6)^2}{12}$
      $3 = \frac{36}{12}$
      $3 = 3$ ✔
      
      The point is on the parabola so it is equidistant to the given point and the given line.
1. Which equation represents the locus of points equidistant from points (1, 1) and (7, 1)?

   (1) $x = 4$ 
   (2) $y = 4$ 
   (3) $x = -4$ 
   (4) $y = -4$

2. Points $A$ and $W$ are 5 inches apart. How many points are equidistant from $A$ and $W$ and also 2 inches from $A$?

   (1) 0 
   (2) 1 
   (3) 2 
   (4) 3

3. The number of points equidistant from two parallel lines and also equidistant from two points on one of the lines is exactly

   (1) 1 
   (2) 2 
   (3) 3 
   (4) 4

4. Point $D$ is on line $PQ$. The locus of the centers of all circles tangent to the line at point $D$ is

   (1) the point $D$ itself. 
   (2) a circle with point $D$ as the center. 
   (3) a line perpendicular to $PQ$ at point $D$. 
   (4) the empty set.

5. What is the locus of points equidistant from the $x$- and $y$-axes and 5 units from the line $y = -3$?

   (1) $y = x$ and $y = -x$ 
   (2) $y = -8$ and $y = 2$ 
   (3) $(-8, -8)$ and $(2, 2)$ 
   (4) $(-8, -8), (-2, 2), (2, 2), (8, -8)$

6. The turning point of the graph of the function $y = 2x^2 + 4x + 3$ is

   (1) $(-1, 1)$ 
   (2) $(-1, -1)$ 
   (3) $(1, -1)$ 
   (4) $(1, 1)$

7. Which is an equation of the axis of symmetry of the graph of the equation $y = x^2 - 6x + 9$?

   (1) $y = 3$ 
   (2) $y = -3$ 
   (3) $x = 3$ 
   (4) $x = -3$

8. Which of the following is the equation of the parabola that is the locus of points equidistant from $F(4, -4)$ and $y = 4$?

   (1) $y = x^2 - 4$ 
   (2) $y = x^2 + 4$ 
   (3) $y = \frac{1}{16}x^2 - \frac{1}{2}x + 1$ 
   (4) $y = -\frac{1}{16}x^2 + \frac{1}{2}x - 1$

9. Write an equation of the locus of points 7 units from the point (1, 3).

   Answer: $(x - 1)^2 + (y - 3)^2 = 49$
10. Write an equation for the locus of points equidistant from the lines whose equations are \( y = 3x - 4 \) and \( y = 3x + 10 \).

**Answer:** \( y = 3x + 3 \)

**Solution:**
The lines \( y = 3x - 4 \) and \( y = 3x + 10 \) are parallel. The \( y \)-intercept, \( b \), of the locus is:

\[
\frac{-4 + 10}{2} = 3
\]

The equation of the locus is \( y = 3x + 3 \).

**PART III**

Answer all questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [8]

11. Find the common solution of the system of equations consisting of \( x^2 + y^2 = 13 \) and \( y = x - 1 \).

**Answer:** \((3, 2)\) and \((-2, -3)\)

**Solution:**
Substitute the second equation for \( y \) in the first equation and solve for \( x \). Then solve for \( y \).

\[
\begin{align*}
x^2 + (x - 1)^2 &= 13 & y &= x - 1 \\
x^2 + x^2 - 2x + 1 &= 13 & y &= 3 - 1 \\
2x^2 - 2x - 12 &= 0 & y &= -2 - 1 \\
x^2 - x - 6 &= 0 & x &= 2 \\
(x - 3)(x + 2) &= 0 & x &= -3
\end{align*}
\]

The solutions are \((3, 2)\) and \((-2, -3)\).

12. *Given:* Right triangle \( ABC \).

Describe the locus of points equidistant from:

a. \( A \) and \( B \).

**Answer:** The perpendicular bisector of \( AB \)

b. the sides \( AC \) and \( AB \).

**Answer:** The bisector of \( \angle A \)

c. \( A, B, \) and \( C \).

**Answer:** The circumcenter of \( \triangle ABC \)
PART IV

Answer all questions in this part. Each correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [12]

13. Write the equation for the locus of points equidistant from the given points \( A(3, 7) \) and \( B(9, 9) \).

Answer: \( y = -3x + 26 \)

Solution:

\[
\text{Midpoint of } AB = \left( \frac{3 + 9}{2}, \frac{7 + 9}{2} \right) = (6, 8)
\]

Slope of \( AB = \frac{9 - 7}{9 - 3} = \frac{2}{6} = \frac{1}{3} \)

Slope of locus = -3

Equation of locus:

\[
\frac{y - 8}{x - 6} = -3
\]

\[
y - 8 = -3x + 18
\]

\[
y = -3x + 26
\]

14. Find the locus of points that are:

a. 1 unit from \( (x - 2)^2 + (y - 2)^2 = 25 \).

Answer: \( (x - 2)^2 + (y - 2)^2 = 16 \) and \( (x - 2)^2 + (y - 2)^2 = 36 \)

b. 2 units from \( y = 4 \).

Answer: \( y = 2 \) and \( y = 6 \)

c. 1 unit from the circle \( (x - 2)^2 + (y - 2)^2 = 25 \), 2 units from \( y = 4 \), and are located inside the given circle.

Answer: \( (6, 2), (-2, 2), (2, 6) \)

Solution:

Intersection of \( y = 2 \) and \( (x - 2)^2 + (y - 2)^2 = 16 \):

\[
(x - 2)^2 + (2 - 2)^2 = 16
\]

\[
x^2 - 4x + 4 = 16
\]

\[
x^2 - 4x - 12 = 0
\]

\[
(x - 6)(x + 2) = 0
\]

\[
x = 6, -2
\]

Intersection of \( y = 6 \) and \( (x - 2)^2 + (6 - 2)^2 = 16 \):

\[
(x - 2)^2 + (6 - 2)^2 = 16
\]

\[
x^2 - 4x + 20 = 16
\]

\[
x^2 - 4x + 4 = 0
\]

\[
(x - 2)(x - 2) = 0
\]

\[
x = 2
\]
1. In $\triangle ABC$ with interior segment $BD$ drawn, $AD = DB = BC$. If $y = 65$, what is the value of $x$?
   
   (1) 32.5  (3) 57.5
   (2) 42.5  (4) 65

2. Which of the following is not sufficient to prove that quadrilateral $ABCD$ is a square?
   
   (1) All four sides are congruent.
   (2) $ABCD$ is a rhombus with one right angle.
   (3) $ABCD$ is a rectangle with two congruent adjacent sides.
   (4) All four angles are right angles and two adjacent sides are congruent.

3. If $k \geq 0$ and points $(k, 2)$ and $(1, k)$ lie on a line with a slope of $k$, then $k =$
   
   (1) 1  (3) 2
   (2) $\sqrt{2}$  (4) $2\sqrt{2}$

4. What is the ratio of the volume of cylinder $A$ with radius $\sqrt{3}$ and height $h$ to the volume of cylinder $B$ with radius $h$ and height $\sqrt{3}$?
   
   (1) 1  (3) $\frac{h}{\sqrt{3}}$
   (2) $\frac{\sqrt{3}}{h}$  (4) $\frac{\pi \sqrt{3}}{h}$

5. In circle $O$, $AC$ is the diameter and $\triangle ABC$ is inscribed in the circle.

   If $PK$ is tangent to the circle at point $C$, then $m\angle BCK =$
   
   (1) $\frac{y}{2}$  (3) $2y$
   (2) $y$  (4) $90 - y$

6. If the statements $\neg (p \land \neg q)$ and $p$ are true, then which statement is also true?
   
   (1) $q$  (3) $\neg p \land q$
   (2) $\neg q$  (4) $\neg p \lor \neg q$

7. The point $(2, 1)$ is the midpoint of a line segment whose endpoints are $(3, 2)$ and $(1, a)$. What is the numerical value of $a$?
   
   (1) 3  (3) 1
   (2) 2  (4) 0

8. In circle $O$, points $A$ and $B$ lie on the circle and equilateral triangle $AOB$ is drawn.

   What is the degree measure of minor arc $\widehat{AB}$?
   
   (1) $30^\circ$  (3) $90^\circ$
   (2) $60^\circ$  (4) $120^\circ
PART II

Answer all questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [4]

9. Find the coordinates of the image of \( P(-6, -4) \) under the composition of the following functions: 
\[
r_{x\text{-axis}} \circ r_{y=x} \circ r_{y\text{-axis}}
\]
Answer: \((-4, -6)\)
Solution:
\[
r_{y\text{-axis}}(-6, -4) = (6, -4)
\]
\[
r_{y=x}(6, -4) = (-4, 6)
\]
\[
r_{x\text{-axis}}(-4, 6) = (-4, -6)
\]

10. Find the volume of a square pyramid with edge \( 2x \) and height \( 3x \).
Answer: \(4x^3\) cu units
Solution:
Area of base = \((2x)^2 = 4x^2\) sq units
\[
V = \frac{1}{3}Bh
\]
\[
= \frac{1}{3}(4x^2)(3x)
\]
\[
= 4x^3\text{ cu units}
\]

PART III

Answer all questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [8]

11. a. The sides of a triangle measure 3, 4, and 5. Find the length of the smallest side of a similar triangle whose perimeter is 8.
Answer: 2
Solution:
Perimeter of given triangle = \(3 + 4 + 5 = 12\)
Let \(x = \) smallest side of the similar triangle
\[
\frac{8}{12} = \frac{x}{3}
\]
\[
24 = 12x
\]
\[
x = 2
\]
Two line segments have lengths 5 and 12. If the length of a third segment is randomly chosen from the set \{5, 7, 8, 11\}, what is the probability that the three segments will form a triangle?

**Answer:** \(\frac{1}{2}\)

**Solution:**
5, 5, 12 do not form a triangle.
7, 5, 12 do not form a triangle.
8, 5, 12 forms a triangle.
11, 5, 12 forms a triangle.

12. In right \(\triangle ABC\), \(\overline{CD}\) is the altitude to hypotenuse \(\overline{AB}\). If \(AD = x\), \(DB = x + 5\), and \(CD = 6\). Find:

a. the value of \(x\).

**Answer:** \(x = 4\)

**Solution:**

\[\frac{AD}{CD} = \frac{CD}{DB}\]
\[\frac{x}{6} = \frac{6}{x+5}\]
\[36 = x^2 + 5x\]
\[0 = x^2 + 5x - 36\]
\[0 = (x + 9)(x - 4)\]
\[x = -9, 4\]

b. the length of \(AC\) in simplest radical form.

**Answer:** \(2\sqrt{13}\)

**Solution:**

\[AD = x = 4\]
\[AB = AD + DB\]
\[= x + x + 5\]
\[= 13\]
\[\frac{AB}{AC} = \frac{AC}{AD}\]
\[\frac{13}{AC} = \frac{AC}{4}\]
\[AC^2 = 52\]
\[AC = \sqrt{52} = 2\sqrt{13}\]
PART IV

Answer all questions in this part. Each correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [12]

13. The vertices of quadrilateral $QRST$ are $Q(a, b), R(0, 0), S(c, 0)$ and $T(a + c, b)$.

Using coordinate geometry, prove that quadrilateral $QRST$ is a parallelogram.

Proof:

Slope of $QR$ is $\frac{b - 0}{a - 0} = \frac{b}{a}$

Slope of $RS$ is $\frac{0 - 0}{c - 0} = 0$

Slope of $ST$ is $\frac{b - 0}{a + c - c} = \frac{b}{a}$

Slope of $TQ$ is $\frac{b - b}{a + c - a} = 0$

$QRST$ contains two pairs of opposite sides that are parallel. Therefore, $QRST$ is a parallelogram.

14. Given: Tangent $\overline{CA}$ and secant $\overline{CDB}$ are drawn to circle $O$, chord $\overline{AB} \equiv \overline{DD}$, and $m\angle CAD = 30$.

Find:

a. $m\overline{AD}$

$m\angle CAD = \frac{1}{2}m\overline{AD}$

$30 = \frac{1}{2}m\overline{AD}$

$m\overline{AD} = 60 \text{ Answer}$

b. $m\angle B$

$m\angle B = \frac{1}{2}m\overline{AD}$

$= \frac{1}{2}(60)$

$= 30 \text{ Answer}$

c. $m\overline{BD}$

Answer: $m\overline{BD} = 150$

Solution:

$m\angle B + 2m\angle BAD = 180 \quad m\angle BAD = \frac{1}{2}m\overline{BD}$

$30 + 2m\angle BAD = 180 \quad 75 = \frac{1}{2}m\overline{BD}$

$2m\angle BAD = 150 \quad m\overline{BD} = 150$

$m\angle BAD = 75$
d. $m\angle BAC$

\[ m\angle BAC = m\angle BAD + m\angle CAD \]
\[ = 30 + 75 \]
\[ = 105 \text{ Answer} \]

e. $m\angle C$

Answer: $m\angle C = 45$

Solution:
\[ \angle ADC = 180 - m\angle ADB \]
\[ = 180 - 75 \]
\[ = 105 \]
\[ m\angle CAD + m\angle ADC + m\angle C = 180 \]
\[ = 30 + 105 + m\angle C = 180 \]
\[ m\angle C = 45 \]