Equations of the Quadratic Type

$$\left(1+\frac{1}{x}\right)^2 - 6\left(1+\frac{1}{x}\right) + 8 = 0$$

Recall: a quadratic is $ax^2+bx+c=0$

$\left(1+\frac{1}{x}\right)=y$

$y^2-6y+8=0$

$(y-4)(y-2)=0$

$y-4=0 \quad y-2=0$

$y=4 \quad y=2$

$\times\left(1+\frac{1}{x}=4\right) \quad \times\left(1+\frac{1}{x}=2\right)$

$x+1=4x \quad x+1=2x$

$1=3x \quad 1=x$

$\frac{1}{3}=x$

Solve for x.

$$x^{\frac{1}{3}} + x^{\frac{1}{6}} - 2 = 0$$

$x^{\frac{2}{6}} + x^{\frac{1}{6}} - 2 = 0$

$\left(x^{\frac{1}{6}}\right)^2 = x^{\frac{2}{6}}$

$\therefore a = x^{\frac{1}{6}}$

$a^2 + a - 2 = 0$

$(a-1)(a+2)=0$

$a-1=0 \quad a+2=0$

$a=1 \quad a=-2$

$\left(x^{\frac{1}{6}}\right)^6=(1)^6 \quad x^{\frac{1}{6}} = -2$

$x=1 \quad x=64$

Solve for x.

$x^6 - 3x^3 - 28 = 0$

$b^2 - 3b - 28 = 0$

$(b-7)(b+4) = 0$

$b - 7 = 0 \quad b + 4 = 0$

$b = 7 \quad b = -4$

$\sqrt[3]{x^3} = \sqrt[3]{7} \quad \sqrt[3]{x^3} = \sqrt[3]{-4}$

$x = \sqrt[3]{7} \quad x = \sqrt[3]{-4}$

$b = x^3$

$(\sqrt[3]{7})^6 - 3(\sqrt[3]{7})^3 - 28 = 0$

$7^{\frac{6}{3}} - 3(7)^{\frac{3}{3}} - 28 = 0$

$7^2 - 21 - 28 = 0$

$49 - 49 = 0 \checkmark$

$(\sqrt[3]{-4})^6 - 3(\sqrt[3]{-4})^3 - 28 = 0$

$(-4)^{\frac{6}{3}} - 3(-4)^{\frac{3}{3}} - 28 = 0$

$(-4)^2 + 12 - 28 = 0$

$16 + 12 - 28 = 0$

$28 - 28 = 0 \checkmark$