Vectors & Projectiles

AP Physics

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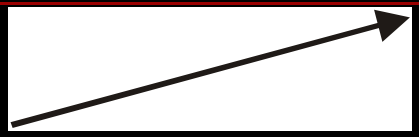
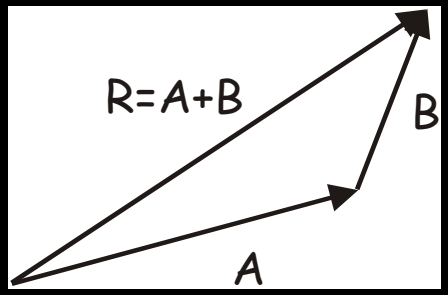
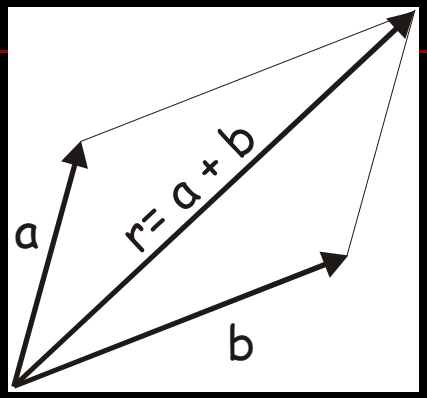
Scalar vs. Vector

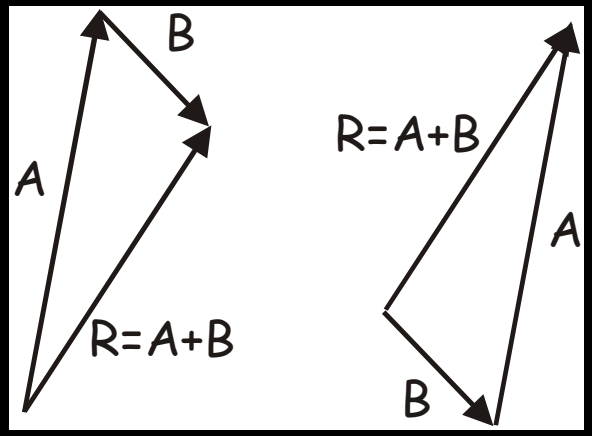
- Scalar - only magnitude
  - Regular Arithmetic applies
    - Ex: Counting Apples
- Vector – has both magnitude and direction
  - Equal Vectors = Same Magnitude & Direction
  - Length represents magnitude

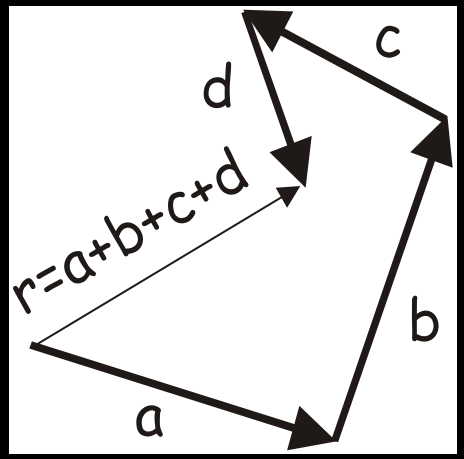
Vectors

- Tail  Head
- Adding Ex:







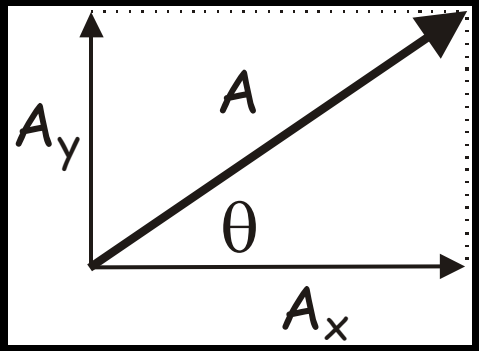


Scale

- Vector is 2.50 cm long, scale is 1 cm = 12.2 m/s, find magnitude
- $2.50\ \cancel{cm}\left(\frac{12.2\ m/s}{1\ \cancel{cm}}\right)$ = 30.5 m/s

Vector Components

- Resolve vector into x and y component vectors



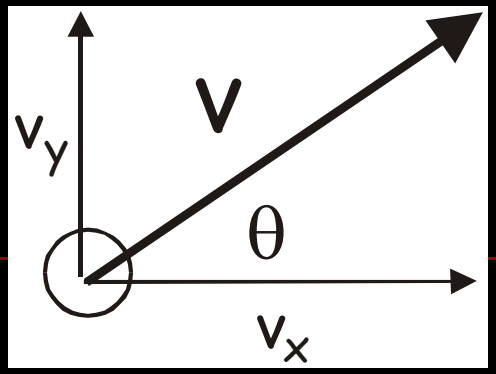
- $A_X = A \cos \theta$ $A_Y = A \sin \theta$ $\theta = \tan^{-1}\left(\frac{A_Y}{A_X}\right)$

- ***Components are independent***

Ex 1:

- A ball is thrown with a velocity of 22.5 m/s at an angle of 23 degrees.
- What is its vertical and horizontal velocity components?
- $v_x = v\cos\theta$ =(22.5 m/s)(cos 23.0°) = 20.1 m/s
- $v_y = v\sin\theta$ =(22.5 m/s)(sin 23.0°) = 8.79 m/s

Ex 2:



- if v_y is 12.4 m/s and v_x is 15.4 m/s, find the angle θ.

$$\theta = \tan^{-1}\left(\frac{v_Y}{v_X}\right)$$

$$\theta = \tan^{-1}\left(\frac{12.4\ m/s}{15.4\ m/s}\right)$$

- = 38.8°

Ex 3:

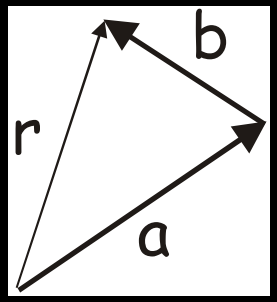
- A ball has a Vx of 12.5 m/s and a Vy of 6.25 m/s. What is its actual velocity and what is the angle of elevation?

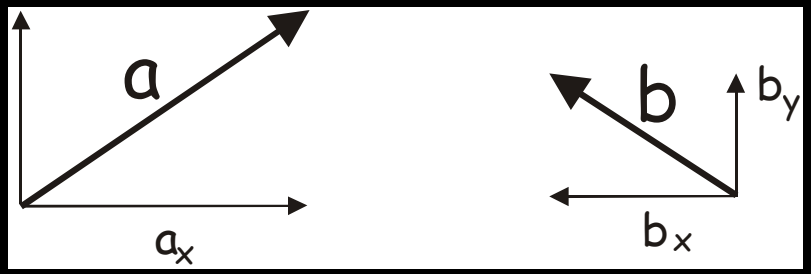
$$v^2 = {v_x}^2 + {v_y}^2 \quad = \sqrt{\left(12.5\frac{m}{s}\right)^2 + \left(6.25\frac{m}{s}\right)^2} \quad = \quad \boxed{14.0\,\frac{m}{s}}$$

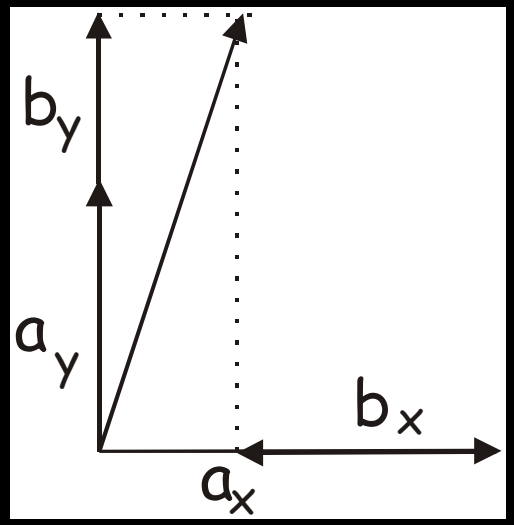
$$\tan\theta = \frac{v_y}{v_x} \quad = \frac{6.25\frac{\cancel{m}}{s}}{12.5\frac{\cancel{m}}{s}} \quad \theta = \quad \boxed{30.0^o}$$

Adding components

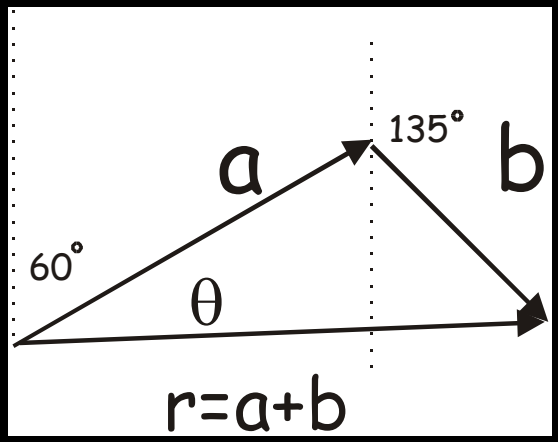
- Find components
  - then
- Add components

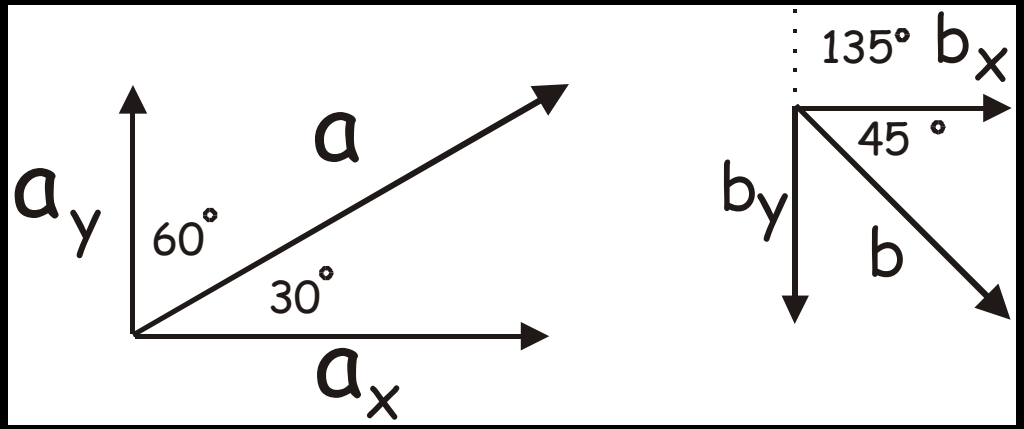






You go hiking. You go 25.0 km in a direction of 060.0°. Then you travel 15.0 km in a direction of 135.0° What is your resultant displacement?





$a_x = a \cos\theta$ = 25.0 m/s cos 30.0° = 21.7 km

$a_y = a \sin\theta$ = 25.0 m/s sin 30.0° = 12.5 km

$b_x = b \cos\theta$ = 15.0 m/s cos 45.0° = 10.6 km

$b_y = b \sin\theta$ = 15.0 m/s sin 45.0° = -10.6 km

- rx = ax + bx
  - = 21.7 km + 10.6 km = 32.3 km
- ry = ay + by
  - = 12.5 km - 10.6 km = 1.9 km

Magnitude:

$$c^2 = a^2 + b^2$$

$$c = \sqrt{a^2 + b^2}$$

- r_x

$$r = \sqrt{(32.3\,km)^2 + (1.9\,km)^2}$$ r = 32.4 km

- Direction:

$$\theta = \tan^{-1}\left(\frac{r_Y}{r_X}\right)$$

$$\theta = \tan^{-1}\left(\frac{1.9\,\cancel{km}}{32.3\,\cancel{km}}\right)$$

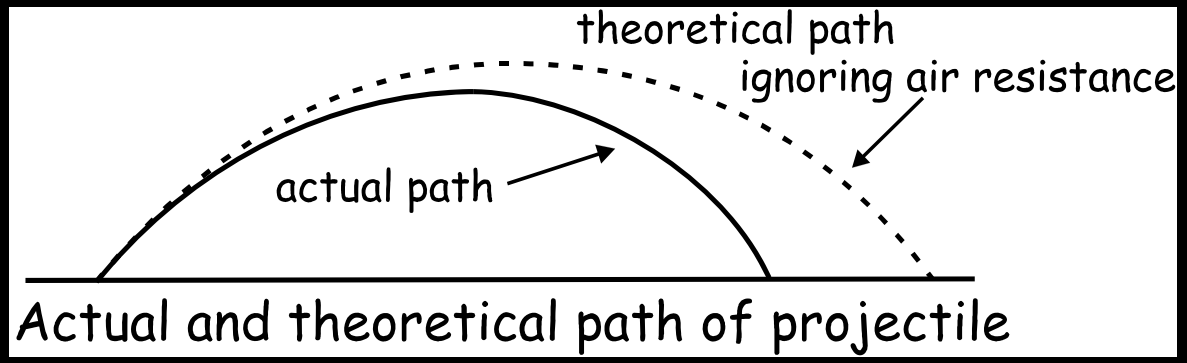
- = 3.37°

Projectiles

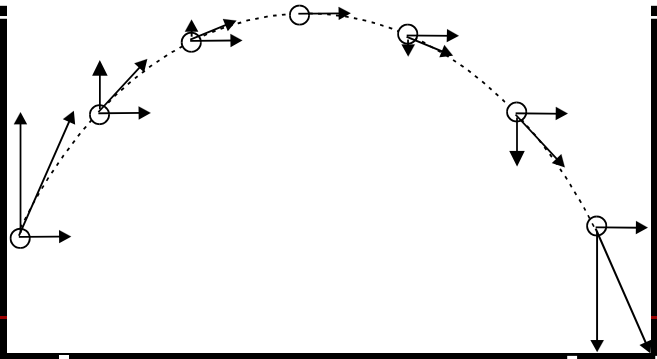
watch for flying cows…

Assumptions:

- g has magnitude of 9.80 m/s^2 and is always downward
- Effect of air resistance can be ignored
- Rotation of earth can be ignored
- ***Path will be parabola***



Actual and theoretical path of projectile



- **perpendicular vectors act independently**
  - $v_{yo} = v0 \sin \theta$
  - $v_{x0} = v0 \cos \theta$
- ***horizontal velocity component constant***
  - *no acceleration in **horizontal direction**.*
- $v = \frac{x}{t}$ ***and*** $v_x = v_{x0} = v_0 \cos \theta_0 =$ ***constant***
- ***Then to find x***
- $x = v_{x0}t = (v_0 \cos \theta_0)t$

Definitions

- v_0 ≡ *initial velocity of projectile*
- v_{y0} ≡ *initial velocity in y direction*
- v_{x0} ≡ v_x = *constant velocity in x direction*
- v ≡ *velocity of projectile*

$$y = v_{y0}t + \frac{1}{2}gt^2 \qquad v_y = v_{y0} + gt \qquad v_y^2 = v_{y0}^2 + 2gy$$

$$y = \frac{1}{2}gt^2 \qquad v_y = gt \qquad v_y^2 = 2gy$$

- Instantaneous speed:

$$v = \sqrt{v_x^{\,2} + v_y^{\,2}}$$

- A flag pole ornament falls off the top of a 25.0m flagpole. How long would it take to hit the ground?

- Solution: $y = \frac{1}{2} g t^2$ $t^2 = \frac{2y}{g}$
- so that $t = \sqrt{\frac{2y}{g}}$
- Plug in the given values:

$$t = \sqrt{\frac{2(-25.0\ \cancel{m})}{-9.80\ \frac{\cancel{m}}{s^2}}}$$

- *t = 2.26 s*