# Gravity & Circular Motion

AP Physics

# History. (read but don't write)

- Sir Isaac Newton is credited with the discovery of gravity.  Now, of ***course*** we know that he didn't ***really*** discover the thing – let's face it, people knew about gravity for as long as there have been people. Well, what Newton did was to ***describe*** gravity, extend its effects from the surface of the earth out into space to explain the behavior of the planets, and to formulate a mathematical law that accurately described the force of gravity between objects with mass.
- Perhaps the greatest success of his theory of gravity was to successfully explain the motion of the heavens – planets, moons, etc…

# Gravity
## (ok you can start to write now)

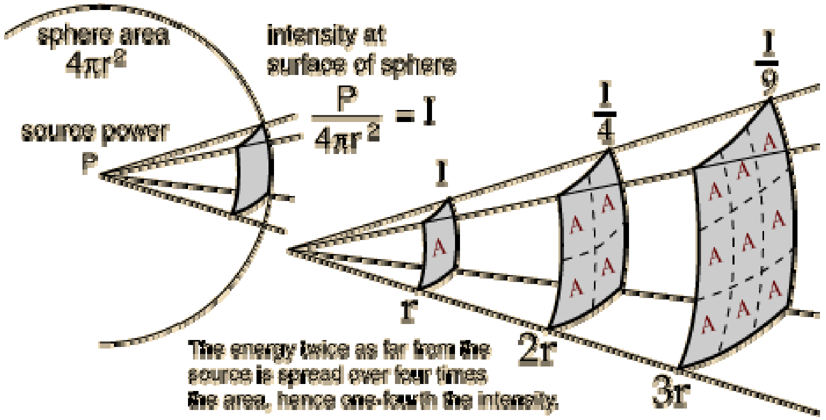
- Gravity is a force of attraction between any two objects that have mass.

$$F = G\frac{m_1 m_2}{r^2}$$

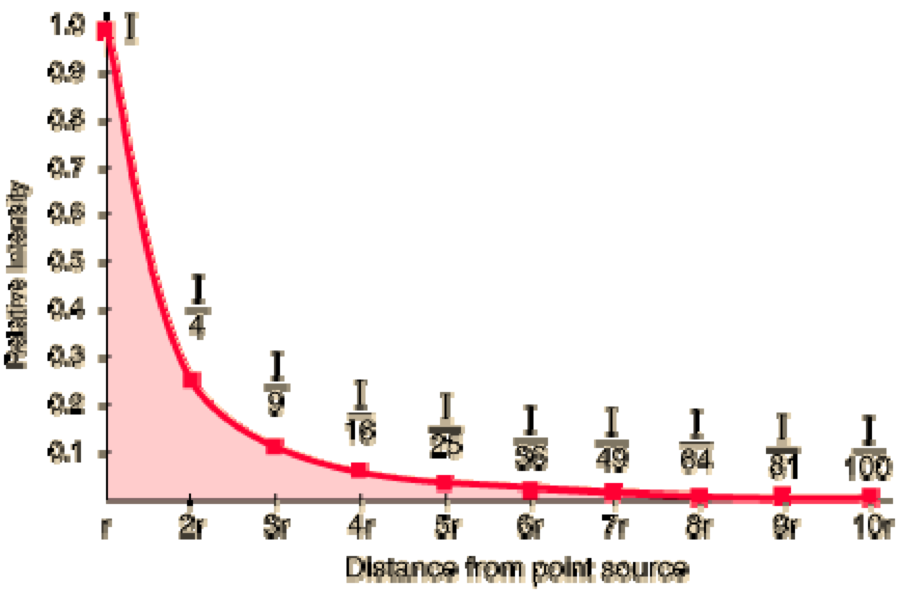
- G ≡ constant of universal gravitation; same everywhere
- $$G = 6.67 \, x \, 10^{-11} \frac{N \cdot m^2}{kg^2}$$

# Inverse Square Law

- This law applies to the weakening of gravity with distance
- The quantity varies as the inverse square of its distance from its source.



# Inverse Square




- The greater the distance from the Earth the less it will weigh.
- No matter how great the distance Earth's gravity does not drop to zero.
- The gravitational influence of every object is exerted through all space.

# Universal Law of Gravitation

- Gravitational force is a field force between two particles -- in all mediums.
- Force varies as the inverse square of the distance
- Force is proportional to mass of objects.
- The gravity force acts from the center of the two objects.
- The gravitational force is always attractive.
- The gravitational force cannot be shielded or canceled.

# Ex:

- A girl, Brandy (42.5 kg), sits 1.50m from a boy (63.0 kg), George. What is the force of gravity between them? (This will tell us how attracted they are to each other.)

$$F = G\frac{m_1 m_2}{r^2} \qquad F = \left(6.67\,x\,10^{-11}\,\frac{N\cdot \cancel{m}^2}{\cancel{kg}^2}\right)\frac{(42.5\,\cancel{kg})(63.0\,\cancel{kg})}{(1.50\,\cancel{m})^2}$$

$$F = 7940\,x\,10^{-11}\,N \quad = \quad \boxed{7.94\,x\,10^{-8}\,N}$$

- As you can see Gravity is a very weak force

# Finding acceleration

- We know: $F = ma$ and $F = G\frac{m_1 m_2}{r^2}$

$$m_2 a = G\frac{m_1 m_2}{r^2}$$

$$a = G\frac{M}{r^2}$$

# Ex:

- Venus has a mass of 4.88*x*10[24] kg and a average radius of 6.06*x*10[6] m. What is the gravitational acceleration on Venus?

$$a = \left(6.67\,x\,10^{-11}\,\frac{kg \cdot m \cdot m^2}{s^2 \cdot kg^2}\right)\frac{\left(4.88\,x\,10^{24}\,kg\right)}{\left(6.06\,x\,10^{6}\,m\right)^2} = 0.886\,x\,10^{1}\,\frac{m}{s^2} = \boxed{8.86\frac{m}{s^2}}$$

- Let’s do Gravity homework #1

# Circular Motion

- Objects moving in a circle undergo constant linear change in direction.

$$angular\ velocity = \frac{angular\ displacement}{time}$$

$$\omega = \frac{\Delta\theta}{\Delta t} \qquad \omega = \frac{\theta}{t}$$

- Measured in: rpm, $\frac{rev}{\min}$ , $\frac{rev}{s}$ , $\frac{rad}{\min}$

# Radian reminder

- (Courtney you don't have to write it if you know it)
- $2\pi$ rad = 360°

$$1\,rad = \frac{360^\circ}{2\pi}$$

- 1 *rad* = 57.3°

- Angular Acceleration

$$angular\ accelertaion = \frac{angular\ velocity}{time}$$

$$\alpha = \frac{\Delta\omega}{\Delta t} \qquad \alpha = \frac{\omega}{t}$$

# *Linear Speed and Circular Motion*

- Time for one rotation – Period

$$\omega = \frac{\theta}{t} \qquad t = \frac{\theta}{\omega}$$

- Let the angular displacement be 1

$$t = \frac{\theta}{\omega} = \frac{1}{\omega}$$

- This is period of the rotation

- equation for speed

$$v = \frac{d}{t}$$

- distance = $2\pi r$

$$v = \frac{d}{t} = \frac{2\pi r}{\frac{1}{\omega}} = 2\pi r\omega$$

- ***linear or tangential speed*** *v* = 2∏*r*$\omega$
- *Or v = r*$\omega$
  - *if* $\omega$ *is in rad/s*

# Centripetal Force

- Whirl ball on string - ball wants to travel in a straight line
- Newton's first law -- inertia
- Direction of force is toward center
- The acceleration toward the center ≡ centripetal acceleration.

$$a_c = \frac{v^2}{r}$$

- Using Newton's 2nd law

$$F_c = ma_c = \frac{mv^2}{r}$$

# Centripetal Force

- "center seeking"
- <u>changes direction</u> of object's velocity vector.
- Ex: 1.2 kg stone is attached to a 1.3 m line and swung in a circle. If it has a linear speed of 13m/s, what is the centripetal force?

$$F_C = ma_C = \frac{mv^2}{r} = \frac{1.2\,kg\left(13\,\frac{m}{s}\right)^2}{1.3\,m} = \boxed{160\,N}$$

# Ex2

- Car has constant speed and makes a turn of radius = 50.0 m. Speed = 15.0 m/s. Find minimum coefficient of friction.
- Frictional force = centripetal force

$$F_C = ma_C \quad = \frac{mv^2}{r}$$

- Frictional force: $f_s = \mu_s N$
- Assume the road is flat, so $N = mg$

$$\mu_s \cancel{m} g = \frac{\cancel{m} v^2}{r}$$

$$\mu_s = \frac{v^2}{gr}$$

$$\mu_s = \left(15.0 \frac{\cancel{m}}{\cancel{s}}\right)^2 \frac{1}{\left(9.8 \frac{\cancel{m}}{\cancel{s}^2}\right) 50.0 \ \cancel{m}} = \boxed{0.459}$$

# Centrifugal force

- = force that seems to be pushing things away from the center of spin during rotation.
- ***The centrifugal force is a fictional force. There is no actual force that is pushing away from the center of a rotating system. It is actually the object's inertia***

# Water in a circle

- Demo-
- The minimum speed for this is called the ***critical velocity***.
- Critical velocity ≡ minimum velocity for an object to travel in vertical circle and maintain its circular path against the force of gravity.
- The critical velocity is given by the formula

$$v_{\min} = \sqrt{gr}$$

- Gravity #3 worksheet, comin’ at ya

# Satellite Motion

- For an object in Orbit
  - The $F_C$ is provided by Gravity
- $$F = G\frac{m_1 m_2}{r^2}$$
- and

$$F_C = \frac{m_2 v^2}{r}$$

- Mass of satellite cancels out

$$G\frac{m_1 m_2}{r^2} = \frac{m_2 v^2}{r}$$

$$v^2 = \frac{G m_1}{r}$$

$$v = \sqrt{\frac{G m_1}{r}}$$

= orbital velocity

# Period of satellite

- $T$ = period $\equiv$ time for one orbit

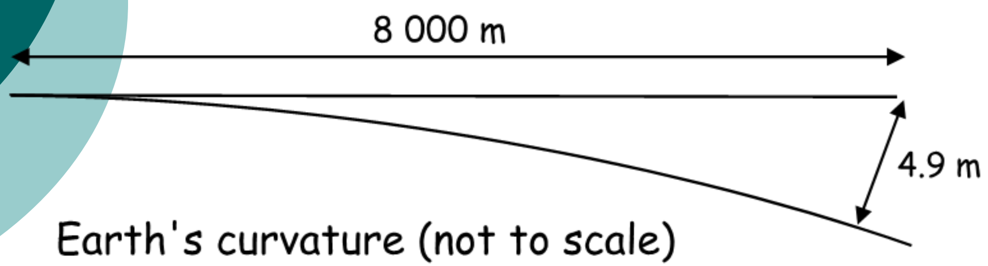
$$v = \frac{x}{t} \qquad t = \frac{x}{v}$$

- $x$ is circumference of orbit $x = 2\pi r$ so $t = \frac{2\pi r}{v}$
- And $v = \sqrt{\frac{Gm_1}{r}}$ combine & simplify…

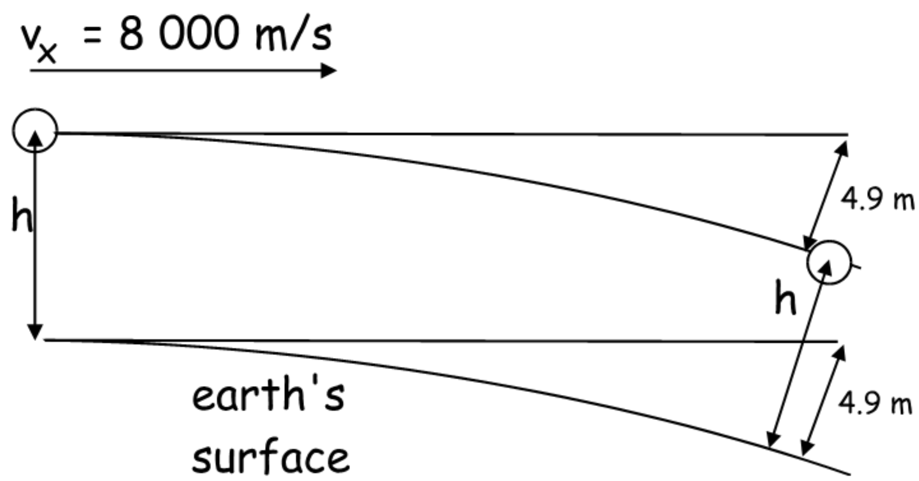
$$t = \frac{2\pi r}{\sqrt{\frac{Gm_1}{r}}}$$

$$\boxed{t = 2\pi \sqrt{\frac{r^3}{Gm_1}}}$$

# Curvature of the Earth




- Earth's surface drops 4.9m in 8000m



# Ex

- A satellite is in a low earth orbit, some 250 km above the earth's surface. $r_{earth}$ is $6.37 \times 10^6$m and $m_{earth} = 5.98 \times 10^{24}$kg. Find period in minutes.

# solution

- *r* is the radius of the earth plus height

$$r = 6.37 \text{ x } 10^6 \text{ m} + 0.25 \text{ x } 10^6 \text{m} = 6.62 \text{ x } 10^6 \text{ m}$$

$$T = 2\pi\sqrt{\frac{r^3}{Gm}} \qquad T = 2\pi\sqrt{\frac{\left(6.62\ x\,10^6\ \cancel{m}\right)^3}{6.67\ x\,10^{-11}\ \frac{\cancel{kg}\ \cancel{m}}{s^2}\ \frac{\cancel{m}^2}{\cancel{kg}^2}\left(5.98\ x\,10^{24}\ \cancel{kg}\right)}}$$

$$T = 5357\ \cancel{s}\left(\frac{1\,\text{min}}{60\ \cancel{s}}\right) \qquad T = 89.3\,\text{min}$$

# *Gravity in Orbit*

- Astronauts appear to be "weightless‟
- distance to the center of the earth is only about 10 % greater
  - force of gravity only slightly less
- They are really falling endlessly around the earth
- ***Weightlessness is caused by being in a constant state of freefall.***

- Tomorrow – Part II question
- Next – Review
- Then - Test