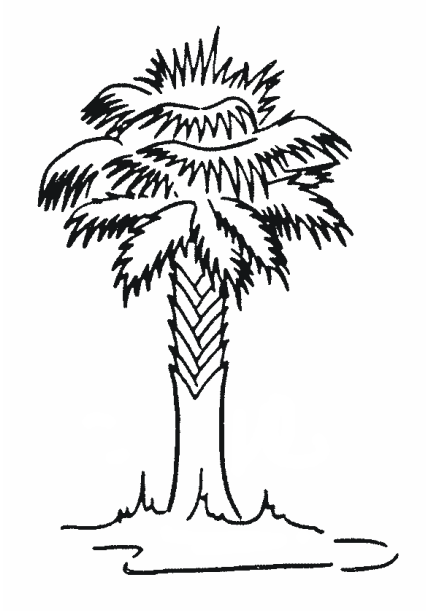
SOUTH CAROLINA ACADEMIC STANDARDS FOR MATHEMATICS



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Contents

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South Carolina Education Oversight Committee
Dr. Jo Anne Anderson, executive director of the South Carolina Education Oversight Committee (EOC), and Dr. Paul Horne, the EOC's director of curriculum and program overview, facilitated the work of three mathematics review teams: a team of mathematics educators from across the nation; a team of South Carolina parents, business, and community leaders; and a team of South Carolina special education and English language learner teachers.

State Mathematics Review Panel
District superintendents recommended educators from around the state to serve as members of the State Mathematics Review Panel. The panel reviewed and recommended revisions to the 2000 standards document, *South Carolina Mathematics Curriculum Standards.*

Mid-Continent Research for Education and Learning
John Kendall, senior director of research at Mid-Continent Research for Education and Learning, led a team of content analysts who provided rigorous, grade-level indicators for the South Carolina standards based on national and state standards documents.

State Department of Education
The mathematics standards in this document were developed under the direction of Lucinda Saylor, deputy superintendent of the Division of Curriculum Services and Assessment, and Dr. Helena Tillar, director of the Office of Curriculum and Standards.

The following State Department of Education staff members assisted in the design and development of this document:

Office of Curriculum and Standards
Dr. John Holton, Coordinator of Mathematics and Science Unit
Mary L. Ruzga, Education Associate, Writing Panel Coordinator
Jeannie Martin, Mathematics Specialist, Elementary Writing Panel Chair
Terrie R. Dew, Mathematics Specialist, Middle Writing Panel Chair
Dr. Heyward Hickman, Education Associate, Secondary Writing Panel Chair

Office of Assessment
Amelia Brailsford, Coordinator of Test Development
Buck Brown, Mathematics Assessment Specialist
Harriet H. Pritchard, Mathematics Assessment Specialist

State Mathematics Writing Panel
Sandy Branyon, Jasper County School District
Gloria H. Brown, Horry County School District
Deborah S. Donovan, Lexington School District Two
Bill Gillam, Richland School District Two
Sandra Goff, Upstate Regional Center for Educational Support
Dr. Paula Gregg, Commission on Higher Education
Mary Beth Heirs, Lexington School District Four
Dianne S. Steelman, Lexington School District One
Rhonda A. Willis, Hampton School District One

Professional Organizations
South Carolina Council of Teachers of Mathematics
South Carolina Leaders of Mathematics Education

Introduction

This document contains the revised academic standards in mathematics for South Carolina students from kindergarten through the twelfth grade. A field review of the first draft of these standards was conducted from October 10 through November 29, 2006. Feedback from that review was incorporated into the final draft, which was presented to the State Board of Education in January 2007.

The *South Carolina Mathematics Academic Standards* is not a curriculum. The academic standards in this document are not sequenced for instruction; do not prescribe classroom activities or materials; and do not dictate instructional strategies, approaches, or practices. A mathematics standards support document, issued by the State Department of Education (SDE), will serve as a resource for districts in constructing district-level standards-based mathematics curricula. By constructing an individual district mathematics curriculum, each district may expand or add topics and organize course content to fit its particular students' needs.

Development and Review of the South Carolina Mathematics Academic Standards

Beginning in 2004, the term for the state-approved expectations for student learning and academic performance in South Carolina was changed from *curriculum standards* to *academic standards.* The SDE, in partnership with Mid-Continent Research for Education and Learning, developed the academic standards and indicators for mathematics by utilizing a number of resources. Central among them were the *South Carolina Mathematics Curriculum Standards 2000*; recommendations made by the State Mathematics Review Panel; and the EOC report containing recommendations from national experts, parents, and business leaders.

The mathematics standards set forth in the *South Carolina Mathematics Curriculum Standards 2000* document were aligned with the national standards published in 2000 by the National Council of Teachers of Mathematics (NCTM) in the document *Principles and Standards for School Mathematics* (available online at http://standards.nctm.org/document/index.htm). Those national standards have also served as a guide for this 2007 edition of the South Carolina academic standards for mathematics and the supporting indicators. The academic standards documents of a number of states as well as the following publications were also utilized:

- *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics* (Reston, VA: NCTM, 2006—available online at http://www.nctm.org/focalpoints/downloads.asp)

- *Mathematics Assessment and Exercise Specifications for the National Assessment of Educational Progress*, developed by the Council of Chief State School Officers, NAEP Mathematics Consensus Project (Washington, DC: National Assessment Governing Board, U.S. Department of Education, n.d.)

- *Mathematics Framework for the 2005 National Assessment of Educational Progress*, developed by the Council of Chief State School Officers, NAEP Mathematics Project (Washington, DC: National Assessment Governing Board, U.S. Department of Education, n.d.)

Procedures for the review of all newly revised South Carolina academic standards, which were agreed upon by the SDE and the EOC, are published in the document *Procedures for the Cyclical Review of Current South Carolina K–12 Academic Standards and for the Development of New Academic Standards.* Those procedures were used in the field review of the first draft of the revised standards document.

Changes in the South Carolina Mathematics Academic Standards Document

The structure and organization of the South Carolina mathematics standards document have been changed in several ways:

- Academic standards are specified for nine grade levels (kindergarten through grade eight) and for five high school core areas (elementary algebra, intermediate algebra, geometry, precalculus, and data analysis and probability).

- Each grade-level and core-area set of standards is now preceded by an overview page. For kindergarten through grade eight, the overview sets forth highlights of new learning. For the high school core areas, the overview provides information concerning the content of the standards with regard to the particular courses that are based on them. The mathematics standards and their indicators describe a connected body of mathematical processes, understandings, and competencies and should serve as the basis for the development of district-level curricula.

- The number of standards—which now ranges from six to seven for each grade or high school core area—has been significantly reduced.

- In kindergarten through grade eight, a standard is written specifically for each of the five mathematical strands—number and operations, algebra, geometry, measurement, and data analysis and probability—to afford a clear vertical articulation of content.

- Each grade level and high school core area begins with the mathematical processes standard, which centers in the specific methods that students will use in applying the skills and knowledge reflected in each five strands that follow this first standard: problem solving, reasoning and proof, communication, connections, and representation.

- The statements of the academic standards are newly constructed. Each of the standards that follow the mathematical processes standard is now stated as one full sentence that begins with the clause "The student will demonstrate through the mathematical processes . . ." and then goes on to specify particular content and skills. The verb "will demonstrate" is used with its general, everyday meaning and does not describe a cognitive category from the taxonomy.

  Following each of the academic standards are indicators, which are intended to help meet the need for grade-level specificity. The indicators are statements of the specific cognitive processes (expressed in the main verbs) and the content knowledge and skills that students must demonstrate in order to meet the grade-level or high school core-area standard.

  The main verbs in the indicators are taxonomic. That is, the main verbs identify specific aspects of the cognitive process as described in the revised Bloom's taxonomy (included in

this standards document as the appendix). Use of the revised taxonomy will allow teachers to identify the kind of content (knowledge) addressed in the indicators (factual, conceptual, procedural, or metacognitive). In addition, use of the revised taxonomy will help teachers align lessons with both the content and the cognitive process identified in the indicators.

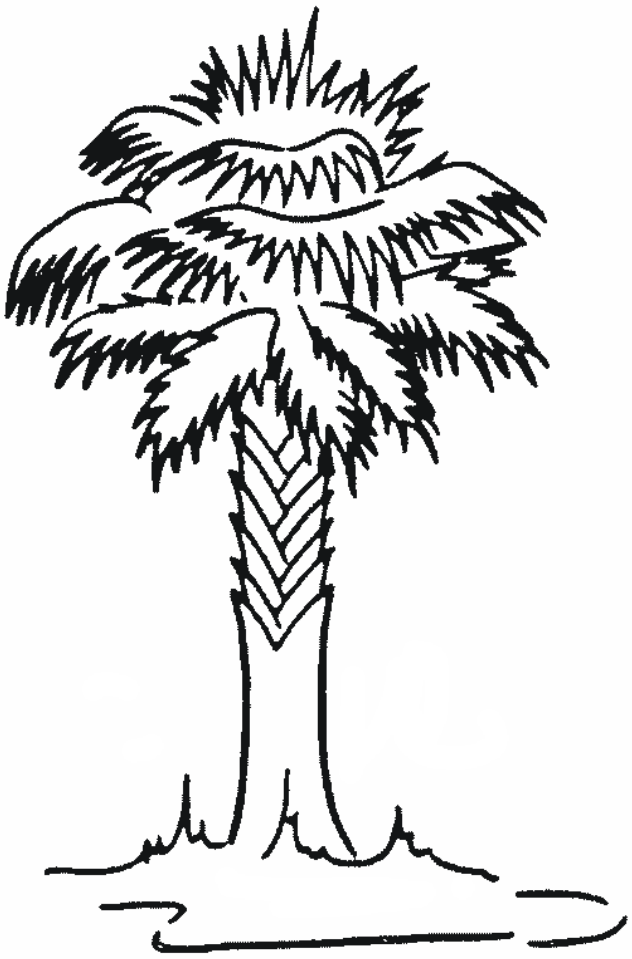
Many of the indicators in mathematics address conceptual knowledge and fall under the second category of cognitive processing, *understanding*, which fosters transfer and meaningful learning rather than rote learning and memorization. These revised mathematics standards also contain some indicators that require students to *analyze* or *evaluate* mathematical representations or situations. As a result, students must use understanding as they demonstrate even more cognitively complex learning.

Statewide Assessments

The mathematics standards and indicators for grades three through eight will be used as the basis for the Palmetto Achievement Challenge Tests (PACT) in mathematics. The mathematics standards for the high school core area of elementary algebra will be used as the basis for items on the state-required end-of-course examination for Algebra 1 and Mathematics for the Technologies 2.

The PACT is based on the broad standards at each grade level. Individual test questions will be aligned with the indicators and in most cases will measure the specific cognitive process stated in the main verb in the indicator. However, some indicators may be assessed through items that address other appropriate cognitive processes within the same category as the main verb in the indicator or may address processes in categories of lower cognitive complexity. For example, the assessment of an indicator that requires students to classify two-dimensional shapes as polygons or nonpolygons—which would fall in the second cognitive category, *understand*—might also ask the student to demonstrate other related cognitive processes such as comparing polygons and nonpolygons or giving examples of polygons or nonpolygons.

GRADE-LEVEL STANDARDS



Kindergarten Overview

This overview provides only the highlights of the new learning that should take place at the kindergarten level. The specific skills and subject matter that kindergartners should be taught in each of the five mathematical strands are set forth in the formal standards and indicators for these strands.

Highlights of the new learning for kindergarteners are

- comparing sets of objects,
- recognizing the effect of addition and subtraction,
- representing place value within specified ranges,
- classifying based on attributes,
- identifying two- and three-dimensional shapes,
- representing basic two-dimensional shapes,
- using positional and directional words to describe location and movement,
- telling time to the hour and using a calendar,
- making nonstandard measurements and identifying measuring devices, and
- organizing data in graphic displays and interpreting data.

KINDERGARTEN

Mathematical Processes

The mathematical processes provide the framework for teaching, learning, and assessing in mathematics at all grade levels. Instructional programs should be built around these processes.

Standard K-1: The student will have a basic understanding of the mathematical processes of problem solving, reasoning and proof, communication, connections, and representation.

The indicators for this standard, which are appropriate for kindergarten through grade two, are adapted from *Principles and Standards for School Mathematics* (NCTM 2000). Classroom application should be based on the standard and its indicators; the mathematical goals for the class; and the skills, needs, and understandings of the particular students.

Indicators

K-1.1 Apply substantive mathematical problem-solving strategies.
K-1.2 Generate conjectures and exchange mathematical ideas.
K-1.3 Explain and justify answers to simple problems.
K-1.4 Analyze patterns by reasoning systematically.
K-1.5 Generalize mathematical concepts.
K-1.6 Use a variety of forms of mathematical communication.
K-1.7 Generalize connections among mathematics, the environment, and other subjects.
K-1.8 Use multiple informal representations to convey mathematical ideas.

KINDERGARTEN

Number and Operations

Standard K-2: The student will demonstrate through the mathematical processes an emerging sense of quantity and numeral relationships, sets, and place values.

Indicators

K-2.1 Recall numbers, counting forward through 99 and backward from 10.
K-2.2 Translate between numeral and quantity through 31.
K-2.3 Compare sets of no more than 31 objects by using the terms *more than, less than,* and *the same as.*
K-2.4 Represent simple joining and separating situations through 10.
K-2.5 Understand that addition results in increase and subtraction results in decrease.
K-2.6 Analyze the magnitude of digits through 99 on the basis of their place values.
K-2.7 Represent the place value of each digit in a two-digit whole number.
K-2.8 Identify ordinal positions through 31st.

KINDERGARTEN

Algebra

Standard K-3: The student will demonstrate through the mathematical processes an emerging sense of repeating and growing patterns and classification based on attributes.

Indicators

K-3.1 Identify simple growing patterns.
K-3.2 Analyze simple repeating and growing relationships to extend patterns.
K-3.3 Translate simple repeating and growing patterns into rules.
K-3.4 Classify objects according to one or more attributes such as color, size, shape, and thickness.

KINDERGARTEN

Geometry

Standard K-4: The student will demonstrate through the mathematical processes an emerging sense of two- and three-dimensional geometric shapes and relative positions in space.

Indicators

K-4.1 Identify the two-dimensional shapes square, circle, triangle, and rectangle and the three-dimensional shapes cube, sphere, and cylinder.

K-4.2 Represent two-dimensional geometric shapes.

K-4.3 Use the positional words *near, far, below, above, beside, next to, across from,* and *between* to describe the location of an object.

K-4.4 Use the directional words *left* and *right* to describe movement.

KINDERGARTEN

Measurement

Standard K-5: The student will demonstrate through the mathematical processes an emerging sense of coin values and the measurement concepts of length, weight, time, and temperature.

Indicators

K-5.1 Identify a penny, a nickel, a dime, a quarter, and a dollar and the value of each.

K-5.2 Compare the lengths of two objects, both directly and indirectly, to order objects according to length.

K-5.3 Use nonstandard units to explore the measurement concepts of length and weight.

K-5.4 Identify rulers, yardsticks, and tape measures as devices used to measure length; scales and balances as devices used to measure weight; calendars and analog and digital clocks as devices used to measure time; and digital and standard thermometers as devices used to measure temperature.

K-5.5 Understand which measure—length, weight, time, or temperature—is appropriate for a given situation.

K-5.6 Use analog and digital clocks to tell time to the hour.

K-5.7 Use a calendar to identify dates, days of the week, and months of the year.

K-5.8 Recall equivalencies associated with time: 7 days = 1 week and 12 months = 1 year.

KINDERGARTEN
Data Analysis and Probability

Standard K-6: The student will demonstrate through the mathematical processes an emerging sense of organizing and interpret data.

Indicators

K-6.1 Organize data in graphic displays in the form of drawings and pictures.
K-6.2 Interpret data in graphic displays in the form of drawings and pictures.

Grade 1

Overview

This overview provides only the highlights of the new learning that should take place at the first-grade level. The specific skills and subject matter that first graders should be taught in each of the five mathematical strands are set forth in the formal standards and indicators for these strands. To alert educators as to when the progression in learning should occur for students in this grade, specific language is used with certain indicators:

- An indicator beginning with the phrase "**Generate strategies**" addresses a concept that is being formally introduced for the first time, and students must therefore be given experiences that foster conceptual understanding.
- An indicator beginning with the phrase "**Apply an algorithm**," "**Apply a procedure**," **"Apply procedures**," or "**Apply formulas**" addresses a concept that has been introduced in a previous grade: students should already have the conceptual understanding, and the goal must now be fluency.
- An indicator beginning with the phrase "**Apply strategies and formulas**" or "**Apply strategies and procedures**" addresses a concept that is being formally introduced for the first time, yet the goal must nonetheless be that students progress to fluency.

Highlights of the new learning for grade-one students are

- representing quantities in word form,
- recalling basic addition and subtraction facts,
- generating strategies to add and subtract without regrouping,
- understanding how patterns relate to addition and subtraction,
- translating patterns into rules,
- classifying two-dimensional shapes as polygons or nonpolygons,
- identifying line symmetry,
- determining the value of a collection of coins,
- generating and using common referents for whole-inch measurements,
- telling time to the half hour and recognizing past and future dates on a calendar,
- using thermometers to measure temperature, and
- using survey questions to generate data and making predictions based on data.

GRADE 1

Mathematical Processes

The mathematical processes provide the framework for teaching, learning, and assessing in mathematics at all grade levels. Instructional programs should be built around these processes.

Standard 1-1: The student will understand and utilize the mathematical processes of problem solving, reasoning and proof, communication, connections, and representation.

The indicators for this standard, which are appropriate for kindergarten through grade two, are adapted from *Principles and Standards for School Mathematics* (NCTM 2000). Classroom application should be based on the standard and its indicators; the mathematical goals for the class; and the skills, needs, and understandings of the particular students.

Indicators

1-1.1 Apply substantive mathematical problem-solving strategies.
1-1.2 Generate conjectures and exchange mathematical ideas.
1-1.3 Explain and justify answers to simple problems.
1-1.4 Analyze patterns by reasoning systematically.
1-1.5 Generalize mathematical concepts.
1-1.6 Use a variety of forms of mathematical communication.
1-1.7 Generalize connections among mathematics, the environment, and other subjects.
1-1.8 Use multiple informal representations to convey mathematical ideas.

GRADE 1

Number and Operations

Standard 1-2: The student will demonstrate through the mathematical processes a sense of quantity and numeral relationships; the relationships among addition, subtraction, and related basic facts; and the connections among numeric, oral, and written-word forms of whole numbers.

Indicators

1-2.1 Translate between numeral and quantity through 100.
1-2.2 Use estimation to determine the approximate number of objects in a set of 20 to 100 objects.
1-2.3 Represent quantities in word form through *ten*.
1-2.4 Recognize whole-number words that correspond to numerals through *twenty*.
1-2.5 Compare whole-number quantities through 100 by using the terms *is greater than, is less than,* and *is equal to*.
1-2.6 Recall basic addition facts through 9 + 9 and corresponding subtraction facts.
1-2.7 Summarize the inverse relationship between addition and subtraction.
1-2.8 Generate strategies to add and subtract without regrouping through two-digit numbers.
1-2.9 Analyze the magnitude of digits through 999 on the basis of their place values.

GRADE 1

Algebra

Standard 1-3: The student will demonstrate through the mathematical processes a sense of numeric patterns, the relationship between addition and subtraction, and change over time.

Indicators

1-3.1 Analyze numeric patterns in addition and subtraction to develop strategies for acquiring basic facts.
1-3.2 Translate patterns into rules for simple addition and subtraction.
1-3.3 Illustrate the commutative property based on basic facts.
1-3.4 Analyze numeric relationships to complete and extend simple patterns.
1-3.5 Classify a number as odd or even.
1-3.6 Classify change over time as quantitative or qualitative.

GRADE 1

Geometry

Standard 1-4: The student will demonstrate through the mathematical processes a sense of two- and three-dimensional geometric shapes, symmetry, and relative positions and directions in space.

Indicators

1-4.1 Identify the three-dimensional geometric shapes prism, pyramid, and cone.
1-4.2 Analyze the two-dimensional shapes circle, square, triangle, and rectangle.
1-4.3 Classify two-dimensional shapes as polygons or nonpolygons.
1-4.4 Identify a line of symmetry.
1-4.5 Use the positional and directional terms *north, south, east,* and *west* to describe location and movement.

GRADE 1

Measurement

Standard 1-5: The student will demonstrate through the mathematical processes a sense of the value of combinations of coins and the measurement of length, weight, time, and temperature.

Indicators

1-5.1 Use a counting procedure to determine the value of a collection of pennies, nickels, dimes, and quarters totaling less than a dollar.
1-5.2 Represent a nickel, a dime, a quarter, a half-dollar, and a dollar in combinations of coins.
1-5.3 Represent money by using the cent and dollar notations.
1-5.4 Use whole-inch units to measure the length of an object.
1-5.5 Generate common referents for whole inches.
1-5.6 Use common referents to make estimates in whole inches.
1-5.7 Use nonstandard units to measure the weight of objects.
1-5.8 Use analog and digital clocks to tell and record time to the half hour.
1-5.9 Illustrate past and future dates on a calendar.
1-5.10 Represent dates in standard form (June 1, 2007, for example) and numeric form (6-1-2007, for example).
1-5.11 Use Celsius and Fahrenheit thermometers to measure temperature.

GRADE 1

Data Analysis and Probability

Standard 1-6: The student will demonstrate through the mathematical processes a sense of collecting, organizing, and interpreting data and of making predictions on the basis of data.

Indicators

1-6.1 Use survey questions to collect data.
1-6.2 Organize data in picture graphs, object graphs, bar graphs, and tables.
1-6.3 Interpret data in picture graphs, object graphs, bar graphs, and tables by using the comparative terms *more*, *less*, *greater*, *fewer*, *greater than*, and *less than*.
1-6.4 Predict on the basis of data whether events are *likely* or *unlikely* to occur.

Grade 2

Overview

This overview provides only the highlights of the new learning that should take place at the second-grade level. The specific skills and subject matter that second graders should be taught in each of the five mathematical strands are set forth in the formal standards and indicators for these strands. To alert educators as to when the progression in learning should occur for students in this grade, specific language is used with certain indicators:

- An indicator beginning with the phrase "**Generate strategies**" addresses a concept that is being formally introduced for the first time, and students must therefore be given experiences that foster conceptual understanding.
- An indicator beginning with the phrase "**Apply an algorithm**," "**Apply a procedure**," **"Apply procedures**," or "**Apply formulas**" addresses a concept that has been introduced in a previous grade: students should already have the conceptual understanding, and the goal must now be fluency.
- An indicator beginning with the phrase "**Apply strategies and formulas**" or "**Apply strategies and procedures**" addresses a concept that is being formally introduced for the first time, yet the goal must nonetheless be that students progress to fluency.

Highlights of the new learning for grade-two students are

- estimating items in a set,
- understanding equal groupings as repeated addition and sharing equally as repeated subtraction,
- generating strategies to add and subtract two-digit numerals with regrouping,
- generating strategies to round numbers to the nearest ten,
- analyzing patterns in skip counting,
- identifying multiple lines of symmetry,
- predicting results of combing and subdividing two-dimensional shapes,
- making change,
- using appropriate tools and units to measure,
- telling time to the nearest five-minute interval and quarter hour,
- matching *a.m.* and *p.m.* to familiar situations,
- creating survey questions to collect data, and
- inferring trends and making predictions based on data sets.

GRADE 2

Mathematical Processes

The mathematical processes provide the framework for teaching, learning, and assessing in mathematics at all grade levels. Instructional programs should be built around these processes.

Standard 2-1: The student will understand and utilize the mathematical processes of problem solving, reasoning and proof, communication, connections, and representation.

The indicators for this standard, which are appropriate for kindergarten through grade two, are adapted from *Principles and Standards for School Mathematics* (NCTM 2000). Classroom application should be based on the standard and its indicators; the mathematical goals for the class; and the skills, needs, and understandings of the particular students.

Indicators

2-1.1 Apply substantive mathematical problem-solving strategies.
2-1.2 Generate conjectures and exchange mathematical ideas.
2-1.3 Explain and justify answers to simple problems.
2-1.4 Analyze patterns by reasoning systematically.
2-1.5 Generalize mathematical concepts.
2-1.6 Use a variety of forms of mathematical communication.
2-1.7 Generalize connections among mathematics, the environment, and other subjects.
2-1.8 Use multiple informal representations to convey mathematical ideas.

GRADE 2

Number and Operations

Standard 2-2: The student will demonstrate through the mathematical processes an understanding of the base-ten numeration system; place values; and accurate, efficient, and generalizable methods of adding and subtracting whole numbers.

Indicators

2-2.1 Generate estimation strategies to determine the approximate number of objects in a set of no more than 1,000 objects.
2-2.2 Represent quantities in word form through *twenty*.
2-2.3 Represent multiples of ten in word form through *ninety*.
2-2.4 Compare whole-number quantities through 999 by using the terms *is less than, is greater than,* and *is equal to* and the symbols <, >, and =.
2-2.5 Interpret models of equal grouping (multiplication) as repeated addition and arrays.
2-2.6 Interpret models of sharing equally (division) in as repeated subtraction and arrays.
2-2.7 Generate strategies to add and subtract pairs of two-digit whole numbers with regrouping.
2-2.8 Generate addition and subtraction strategies to find missing addends and subtrahends in number combinations through 20.
2-2.9 Generate strategies to round numbers through 90 to the nearest 10.
2-2.10 Analyze the magnitude of digits through 9,999 on the basis of their place values.

GRADE 2

Algebra

Standard 2-3: The student will demonstrate through the mathematical processes an understanding of numeric patterns and quantitative and qualitative change.

Indicators

2-3.1 Analyze numeric patterns in skip counting that uses the numerals 1 through 10.
2-3.2 Translate patterns into rules for simple multiples.
2-3.3 Analyze relationships to complete and extend growing and repeating patterns involving numbers, symbols, and objects.
2-3.4 Identify quantitative and qualitative change over time.
2-3.5 Analyze quantitative and qualitative change over time.

GRADE 2

Geometry

Standard 2-4: The student will demonstrate through the mathematical processes an understanding of basic spatial reasoning and the connection between the identification of basic attributes and the classification of three-dimensional shapes.

Indicators

2-4.1 Analyze the three-dimensional shapes spheres, cubes, cylinders, prisms, pyramids, and cones according to the number and shape of the faces, edges, corners, and bases of each.
2-4.2 Identify multiple lines of symmetry.
2-4.3 Predict the results of combining and subdividing polygons and circles.

GRADE 2

Measurement

Standard 2-5: The student will demonstrate through the mathematical processes an understanding of the value of combinations of coins and bills and the measurement of length, weight, time, and temperature.

Indicators

2-5.1 Use a counting procedure to determine the value of a collection of coins and bills.
2-5.2 Use coins to make change up to one dollar.
2-5.3 Use appropriate tools to measure objects to the nearest whole unit: measuring length in centimeters, feet, and yards; measuring liquid volume in cups, quarts, and gallons; measuring weight in ounces and pounds; and measuring temperature on Celsius and Fahrenheit thermometers.
2-5.4 Generate common measurement referents for feet, yards, and centimeters.
2-5.5 Use common measurement referents to make estimates in feet, yards, and centimeters.
2-5.6 Predict whether the measurement will be greater or smaller when different units are used to measure the same object.
2-5.7 Use analog and digital clocks to tell and record time to the nearest quarter hour and to the nearest five-minute interval.
2-5.8 Match *a.m.* and *p.m.* to familiar situations.
2-5.9 Recall equivalencies associated with length and time: 12 inches = 1 foot, 3 feet = 1 yard, 60 minutes = 1 hour, and 24 hours = 1 day.

GRADE 2

Data Analysis and Probability

Standard 2-6: The student will demonstrate through the mathematical processes an understanding of creating questions to collect data, organizing data, describing trends of a data set, and making predictions based on data.

Indicators

2-6.1 Create survey questions to collect data.
2-6.2 Organize data in charts, pictographs, and tables.
2-6.3 Infer trends in a data set as increasing, decreasing, or random.
2-6.4 Predict on the basis of data whether events are *more likely* or *less likely* to occur.

Grade 3

Overview

This overview provides only the highlights of the new learning that should take place at the third-grade level. The specific skills and subject matter that third graders should be taught in each of the five mathematical strands are set forth in the formal standards and indicators for these strands. To alert educators as to when the progression in learning should occur for students in this grade, specific language is used with certain indicators:

- An indicator beginning with the phrase "**Generate strategies**" addresses a concept that is being formally introduced for the first time, and students must therefore be given experiences that foster conceptual understanding.
- An indicator beginning with the phrase "**Apply an algorithm**," "**Apply a procedure**," **"Apply procedures**," or "**Apply formulas**" addresses a concept that has been introduced in a previous grade: students should already have the conceptual understanding, and the goal must now be fluency.
- An indicator beginning with the phrase "**Apply strategies and formulas**" or "**Apply strategies and procedures**" addresses a concept that is being formally introduced for the first time, yet the goal must nonetheless be that students progress to fluency.

Highlights of the new learning for grade-three students are

- symbolically comparing number quantities;
- applying an algorithm to add and subtract whole numbers fluently;
- applying the concept of fractions;
- recalling basic multiplication and division facts;
- generating strategies to multiply one single-digit whole-number factor and one double-digit whole-number factor;
- using symbols to represent an unknown quantity in a simple addition, subtraction, or multiplication equation;
- understanding the attributes of circles;
- classifying polygons;
- classifying lines, line segments, and angles;
- predicting the results of one transformation;
- generating strategies to determine perimeters of polygons;
- telling time to the nearest minute;
- applying a procedure to find the range of a data set;
- comparing the benefits of multiple representations of a given data set; and
- understanding when the probability of an event is 0 or 1.

GRADE 3

Mathematical Processes

The mathematical processes provide the framework for teaching, learning, and assessing in mathematics at all grade levels. Instructional programs should be built around these processes.

Standard 3-1: The student will understand and utilize the mathematical processes of problem solving, reasoning and proof, communication, connections, and representation.

The indicators for this standard, which are appropriate for grades three through five, are adapted from *Principles and Standards for School Mathematics* (NCTM 2000). Classroom application should be based on the standard and its indicators; the mathematical goals for the class; and the skills, needs, and understandings of the particular students.

Indicators

3-1.1 Analyze information to solve increasingly more sophisticated problems.
3-1.2 Construct arguments that lead to conclusions about general mathematical properties and relationships.
3-1.3 Explain and justify answers on the basis of mathematical properties, structures, and relationships.
3-1.4 Generate descriptions and mathematical statements about relationships between and among classes of objects.
3-1.5 Use correct, complete, and clearly written and oral mathematical language to pose questions, communicate ideas, and extend problem situations.
3-1.6 Generalize connections between new mathematical ideas and related concepts and subjects that have been previously considered.
3-1.7 Use flexibility in mathematical representations.
3-1.8 Recognize the limitations of various forms of mathematical representations.

GRADE 3

Number and Operations

Standard 3-2: The student will demonstrate through the mathematical processes an understanding of the representation of whole numbers and fractional parts; the addition and subtraction of whole numbers; accurate, efficient, and generalizable methods of multiplying whole numbers; and the relationships among multiplication, division, and related basic facts.

Indicators

3-2.1 Compare whole-number quantities through 999,999 by using the terms *is less than, is greater than*, and *is equal to* and the symbols <, >, and =.
3-2.2 Represent in word form whole numbers through *nine hundred ninety-nine thousand.*
3-2.3 Apply an algorithm to add and subtract whole numbers fluently.
3-2.4 Apply procedures to round any whole number to the nearest 10, 100, or 1,000.
3-2.5 Understand fractions as parts of a whole.
3-2.6 Represent fractions that are greater than or equal to 1.
3-2.7 Recall basic multiplication facts through 12 x 12 and the corresponding division facts.
3-2.8 Compare the inverse relationship between multiplication and division.
3-2.9 Analyze the effect that adding, subtracting, or multiplying odd and/or even numbers has on the outcome.
3-2.10 Generate strategies to multiply whole numbers by using one single-digit factor and one multidigit factor.
3-2.11 Use basic number combinations to compute related multiplication problems that involve multiples of 10.
3-2.12 Analyze the magnitude of digits through 999,999 on the basis of their place value.

GRADE 3

Algebra

Standard 3-3: The student will demonstrate through the mathematical processes an understanding of numeric patterns, symbols as representations of unknown quantity, and situations showing increase over time.

Indicators

3-3.1 Create numeric patterns that involve whole-number operations.
3-3.2 Apply procedures to find missing numbers in numeric patterns that involve whole-number operations.
3-3.3 Use symbols to represent an unknown quantity in a simple addition, subtraction, or multiplication equation.
3-3.4 Illustrate situations that show change over time as increasing.

GRADE 3

Geometry

Standard 3-4: The student will demonstrate through the mathematical processes an understanding of the connection between the identification of basic attributes and the classification of two-dimensional shapes.

Indicators

3-4.1 Identify the specific attributes of circles: center, radius, circumference, and diameter.
3-4.2 Classify polygons as either triangles, quadrilaterals, pentagons, hexagons, or octagons according to the number of their sides.
3-4.3 Classify lines and line segments as either parallel, perpendicular, or intersecting.
3-4.4 Classify angles as either right, acute, or obtuse.
3-4.5 Classify triangles by the length of their sides as either scalene, isosceles, or equilateral and by the size of their angles as either acute, obtuse, or right.
3-4.6 Exemplify points, lines, line segments, rays, and angles.
3-4.7 Analyze the results of combining and subdividing circles, triangles, quadrilaterals, pentagons, hexagons, and octagons.
3-4.8 Predict the results of one transformation—either slide, flip, or turn—of a geometric shape.

GRADE 3

Measurement

Standard 3-5: The student will demonstrate through the mathematical processes an understanding of length, time, weight, and liquid volume measurements; the relationships between systems of measure; accurate, efficient, and generalizable methods of determining the perimeters of polygons; and the values and combinations of coins required to make change.

Indicators

3-5.1 Use the fewest possible number of coins when making change.
3-5.2 Use appropriate tools to measure objects to the nearest unit: measuring length in meters and half inches; measuring liquid volume in fluid ounces, pints, and liters; and measuring mass in grams.
3-5.3 Recognize the relationship between meters and yards, kilometers and miles, liters and quarts, and kilograms and pounds.
3-5.4 Use common referents to make comparisons and estimates associated with length, liquid volume, and mass and weight: meters compared to yards, kilometers to miles, liters to quarts, and kilograms to pounds.
3-5.5 Generate strategies to determine the perimeters of polygons.
3-5.6 Use analog and digital clocks to tell time to the nearest minute.
3-5.7 Recall equivalencies associated with time and length: 60 seconds = 1 minute and 36 inches = 1 yard.

GRADE 3

Data Analysis and Probability

Standard 3-6: The student will demonstrate through the mathematical processes an understanding of organizing, interpreting, analyzing and making predictions about data, the benefits of multiple representations of a data set, and the basic concepts of probability.

Indicators

3-6.1 Apply a procedure to find the range of a data set.
3-6.2 Organize data in tables, bar graphs, and dot plots.
3-6.3 Interpret data in tables, bar graphs, pictographs, and dot plots.
3-6.4 Analyze dot plots and bar graphs to make predictions about populations.
3-6.5 Compare the benefits of using tables, bar graphs, and dot plots as representations of a given data set.
3-6.6 Predict on the basis of data whether events are *likely*, *unlikely*, *certain*, or *impossible* to occur.
3-6.7 Understand when the probability of an event is 0 or 1.

Grade 4

Overview

This overview provides only the highlights of the new learning that should take place at the fourth-grade level. The specific skills and subject matter that fourth graders should be taught in each of the five mathematical strands are set forth in the formal standards and indicators for these strands. To alert educators as to when the progression in learning should occur for students in this grade, specific language is used with certain indicators:

- An indicator beginning with the phrase "**Generate strategies**" addresses a concept that is being formally introduced for the first time, and students must therefore be given experiences that foster conceptual understanding.
- An indicator beginning with the phrase "**Apply an algorithm**," "**Apply a procedure**," **"Apply procedures**," or "**Apply formulas**" addresses a concept that has been introduced in a previous grade: students should already have the conceptual understanding, and the goal must now be fluency.
- An indicator beginning with the phrase "**Apply strategies and formulas**" or "**Apply strategies and procedures**" addresses a concept that is being formally introduced for the first time, yet the goal must nonetheless be that students progress to fluency.

Highlights of the new learning for grade-four students are

- applying an algorithm to multiply whole numbers fluently;
- generating strategies to divide whole numbers by single-digit divisors;
- applying strategies and procedures to find equivalent forms of fractions and comparing fractions and decimals;
- generating strategies to add and subtract decimals through hundredths;
- translating among letters, symbols, and words to represent quantities in a simple mathematical expression or equation;
- applying procedures to find the value of an unknown in a simple whole-number equation;
- analyzing quadrilaterals;
- predicting results of multiple transformations;
- finding points in the first quadrant of a coordinate grid;
- generating strategies to determine area of rectangles and triangles;
- using equivalencies to convert units of measure within the U.S. Customary System;
- applying strategies and procedures to determine elapsed time within a 12-hour period;
- interpreting data in graphic displays with increments greater or equal to one; and
- analyzing possible outcomes for a simple event.

GRADE 4

Mathematical Processes

The mathematical processes provide the framework for teaching, learning, and assessing in mathematics at all grade levels. Instructional programs should be built around these processes.

Standard 4-1: The student will understand and utilize the mathematical processes of problem solving, reasoning and proof, communication, connections, and representation.

The indicators for this standard, which are appropriate for grades three through five, are adapted from *Principles and Standards for School Mathematics* (NCTM 2000). Classroom application should be based on the standard and its indicators; the mathematical goals for the class; and the skills, needs, and understandings of the particular students.

Indicators

4-1.1 Analyze information to solve increasingly more sophisticated problems.
4-1.2 Construct arguments that lead to conclusions about general mathematical properties and relationships.
4-1.3 Explain and justify answers to problems on the basis of mathematical properties, structures, and relationships on mathematical properties, structures, and relationships.
4-1.4 Generate descriptions and mathematical statements about relationships between and among classes of objects.
4-1.5 Use correct, complete, and clearly written and oral mathematical language to pose questions, communicate ideas, and extend problem situations.
4-1.6 Generalize connections between new mathematical ideas and related concepts and subjects that have been previously considered.
4-1.7 Use flexibility in mathematical representations.
4-1.8 Recognize the limitations of various forms of mathematical representations.

GRADE 4

Number and Operations

Standard 4-2: The student will demonstrate through the mathematical processes an understanding of decimal notation as an extension of the place-value system; the relationship between fractions and decimals; the multiplication of whole numbers; and accurate, efficient, and generalizable methods of dividing whole numbers, adding decimals, and subtracting decimals.

Indicators

4-2.1 Recognize the period in the place-value structure of whole numbers: units, thousands, millions, and billions.

4-2.2 Apply divisibility rules for 2, 5, and 10.

4-2.3 Apply an algorithm to multiply whole numbers fluently.

4-2.4 Explain the effect on the product when one of the factors is changed.

4-2.5 Generate strategies to divide whole numbers by single-digit divisors.

4-2.6 Analyze the magnitude of digits through hundredths on the basis of their place value.

4-2.7 Compare decimals through hundredths by using the terms *is less than, is greater than,* and *is equal to* and the symbols <, >, and =.

4-2.8 Apply strategies and procedures to find equivalent forms of fractions.

4-2.9 Compare the relative size of fractions to the benchmarks 0, $\frac{1}{2}$, and 1.

4-2.10 Identify common the fraction/decimal equivalents $\frac{1}{2}$ =.5, $\frac{1}{4}$ =.25, $\frac{3}{4}$ =.75, $\frac{1}{3} \approx .33$, $\frac{2}{3} \approx .67$, multiples of $\frac{1}{10}$, and multiples of $\frac{1}{100}$.

4-2.11 Represent improper fractions, mixed numbers, and decimals.

4-2.12 Generate strategies to add and subtract decimals through hundredths.

GRADE 4

Algebra

Standard 4-3: The student will demonstrate through the mathematical processes an understanding of numeric and nonnumeric patterns, the representation of simple mathematical relationships, and the application of procedures to find the value of an unknown.

Indicators

4-3.1 Analyze numeric, nonnumeric, and repeating patterns involving all operations and decimal patterns through hundredths.

4-3.2 Generalize a rule for numeric, nonnumeric, and repeating patterns involving all operations.

4-3.3 Use a rule to complete a sequence or a table.

4-3.4 Translate among, letters, symbols, and words to represent quantities in simple mathematical expressions or equations.

4-3.5 Apply procedures to find the value of an unknown letter or symbol in a whole-number equation.

4-3.6 Illustrate situations that show change over time as either increasing, decreasing, or varying.

GRADE 4

Geometry

Standard 4-4: The student will demonstrate through the mathematical processes an understanding of the relationship between two- and three-dimensional shapes, the use of transformations to determine congruency, and the representation of location and movement within the first quadrant of a coordinate system.

Indicators

4-4.1 Analyze the quadrilaterals squares, rectangles, trapezoids, rhombuses, and parallelograms according to their properties.

4-4.2 Analyze the relationship between three-dimensional geometric shapes in the form of cubes, rectangular prisms, and cylinders and their two-dimensional nets.

4-4.3 Predict the results of multiple transformations of the same type—translation, reflection, or rotation—on a two-dimensional geometric shape.

4-4.4 Represent the two-dimensional shapes trapezoids, rhombuses, and parallelograms and the three-dimensional shapes cubes, rectangular prisms, and cylinders.

4-4.5 Use transformation(s) to prove congruency.

4-4.6 Represent points, lines, line segments, rays, angles, and polygons.

4-4.7 Represent with ordered pairs of whole numbers the location of points in the first quadrant of a coordinate grid.

4-4.8 Illustrate possible paths from one point to another along vertical and horizontal grid lines in the first quadrant of the coordinate plane.

GRADE 4

Measurement

Standard 4-5: The student will demonstrate through the mathematical processes an understanding of elapsed time; conversions within the U.S. Customary System; and accurate, efficient, and generalizable methods of determining area.

Indicators

4-5.1 Use appropriate tools to measure objects to the nearest unit: measuring length in quarter inches, centimeters, and millimeters; measuring liquid volume in cups, quarts, and liters; and measuring weight and mass in pounds, milligrams, and kilograms.

4-5.2 Compare angle measures with referent angles of 45 degrees, 90 degrees, and 180 degrees to estimate angle measures.

4-5.3 Use equivalencies to convert units of measure within the U.S. Customary System: converting length in inches, feet, yards, and miles; converting weight in ounces, pounds, and tons; converting liquid volume in cups, pints, quarts, and gallons; and converting time in years, months, weeks, days, hours, minutes, and seconds.

4-5.4 Analyze the perimeter of a polygon.

4-5.5 Generate strategies to determine the area of rectangles and triangles.

4-5.6 Apply strategies and procedures to determine the amount of elapsed time in hours and minutes within a 12-hour period, either a.m. or p.m.

4-5.7 Use Celsius and Fahrenheit thermometers to determine temperature changes during time intervals.

4-5.8 Recall equivalencies associated with liquid volume, time, weight, and length: 8 liquid ounces = 1 cup, 2 cups = 1 pint, 2 pints = 1 quart, 4 quarts = 1 gallon; 365 days = 1 year, 52 weeks = 1 year; 16 ounces = 1 pound, 2,000 pounds = 1 ton; and 5,280 feet = 1 mile.

4-5.9 Exemplify situations in which highly accurate measurements are required.

GRADE 4

Data Analysis and Probability

Standard 4-6: The student will demonstrate through the mathematical processes an understanding of the impact of data-collection methods, the appropriate graph for categorical or numerical data, and the analysis of possible outcomes for a simple event.

Indicators

4-6.1 Compare how data-collection methods impact survey results.
4-6.2 Interpret data in tables, line graphs, bar graphs, and double bar graphs whose scale increments are greater than or equal to 1.
4-6.3 Organize data in tables, line graphs, and bar graphs whose scale increments are greater than or equal to 1.
4-6.4 Distinguish between categorical and numerical data.
4-6.5 Match categorical and numerical data to appropriate graphs.
4-6.6 Predict on the basis of data whether events are *likely, unlikely, certain, impossible,* or *equally likely* to occur.
4-6.7 Analyze possible outcomes for a simple event.

Grade 5

Overview

This overview provides only the highlights of the new learning that should take place at the fifth-grade level. The specific skills and subject matter that fifth graders should be taught in each of the five mathematical strands are set forth in the formal standards and indicators for these strands. To alert educators as to when the progression in learning should occur for students in this grade, specific language is used with certain indicators:

- An indicator beginning with the phrase "**Generate strategies**" addresses a concept that is being formally introduced for the first time, and students must therefore be given experiences that foster conceptual understanding.
- An indicator beginning with the phrase "**Apply an algorithm**," "**Apply a procedure**," **"Apply procedures**," or "**Apply formulas**" addresses a concept that has been introduced in a previous grade: students should already have the conceptual understanding, and the goal must now be fluency.
- An indicator beginning with the phrase "**Apply strategies and formulas**" or "**Apply strategies and procedures**" addresses a concept that is being formally introduced for the first time, yet the goal must nonetheless be that students progress to fluency.

Highlights of the new learning for grade-five students are

- applying an algorithm to divide whole numbers fluently,
- understanding the concept of prime and composite numbers,
- generating strategies to add and subtract fractions,
- applying an algorithm to add and subtract decimals through thousandths,
- classifying shapes as congruent,
- translating between two-dimensional representations and three-dimensional objects,
- predicting results of combined multiple transformations,
- analyzing shapes for line and/or rotational symmetry,
- using a protractor to measure angles,
- using equivalencies to convert units of measure within the metric system,
- applying formulas to determine perimeter and area,
- applying strategies and formulas to determine volume,
- applying procedures to determine elapsed time within a 24-hour period,
- applying procedures to calculate the measures of central tendency, and
- concluding why the sum of the probabilities of the outcomes of an experiment must equal 1.

GRADE 5

Mathematical Processes

The mathematical processes provide the framework for teaching, learning, and assessing in mathematics at all grade levels. Instructional programs should be built around these processes.

Standard 5-1: The student will understand and utilize the mathematical processes of problem solving, reasoning and proof, communication, connections, and representation.

The indicators for this standard, which are appropriate for grades three through five, are adapted from *Principles and Standards for School Mathematics* (NCTM 2000). Classroom application should be based on the standard and its indicators; the mathematical goals for the class; and the skills, needs, and understandings of the particular students.

Indicators

5-1.1 Analyze information to solve increasingly more sophisticated problems.
5-1.2 Construct arguments that lead to conclusions about general mathematical properties and relationships.
5-1.3 Explain and justify answers based on mathematical properties, structures, and relationships.
5-1.4 Generate descriptions and mathematical statements about relationships between and among classes of objects.
5-1.5 Use correct, clear, and complete oral and written mathematical language to pose questions, communicate ideas, and extend problem situations.
5-1.6 Generalize connections between new mathematical ideas and related concepts and subjects that have been previously considered.
5-1.7 Use flexibility in mathematical representations.
5-1.8 Recognize the limitations of various forms of mathematical representations.

GRADE 5

Number and Operations

Standard 5-2: The student will demonstrate through the mathematical processes an understanding of the place value system; the division of whole numbers; the addition and subtraction of decimals; the relationships among whole numbers, fractions, and decimals; and accurate, efficient, and generalizable methods of adding and subtracting fractions.

Indicators

5-2.1 Analyze the magnitude of a digit on the basis of its place value, using whole numbers and decimal numbers through thousandths.
5-2.2 Apply an algorithm to divide whole numbers fluently.
5-2.3 Understand the relationship among the divisor, dividend, and quotient.
5-2.4 Compare whole numbers, decimals, and fractions by using the symbols <, >, and =.
5-2.5 Apply an algorithm to add and subtract decimals through thousandths.
5-2.6 Classify numbers as prime, composite, or neither.
5-2.7 Generate strategies to find the greatest common factor and the least common multiple of two whole numbers.
5-2.8 Generate strategies to add and subtract fractions with like and unlike denominators.
5-2.9 Apply divisibility rules for 3, 6, and 9.

GRADE 5

Algebra

Standard 5-3: The student will demonstrate through the mathematical processes an understanding of the use of patterns, relations, functions, models, structures, and algebraic symbols to represent quantitative relationships and will analyze change in various contexts.

Indicators

5-3.1 Represent numeric, algebraic, and geometric patterns in words, symbols, algebraic expressions, and algebraic equations.
5-3.2 Analyze patterns and functions with words, tables, and graphs.
5-3.3 Match tables, graphs, expressions, equations, and verbal descriptions of the same problem situation.
5-3.4 Identify applications of commutative, associative, and distributive properties with whole numbers.
5-3.5 Analyze situations that show change over time.

GRADE 5

Geometry

Standard 5-4: The student will demonstrate through the mathematical processes an understanding of congruency, spatial relationships, and relationships among the properties of quadrilaterals.

Indicators

5-4.1 Apply the relationships of quadrilaterals to make logical arguments about their properties.
5-4.2 Compare the angles, side lengths, and perimeters of congruent shapes.
5-4.3 Classify shapes as congruent.
5-4.4 Translate between two-dimensional representations and three-dimensional objects.
5-4.5 Predict the results of multiple transformations on a geometric shape when combinations of translation, reflection, and rotation are used.
5-4.6 Analyze shapes to determine line symmetry and/or rotational symmetry.

GRADE 5

Measurement

Standard 5-5: The student will demonstrate through the mathematical processes an understanding of the units and systems of measurement and the application of tools and formulas to determine measurements.

Indicators

5-5.1 Use appropriate tools and units to measure objects to the precision of one-eighth inch.
5-5.2 Use a protractor to measure angles from 0 to 180 degrees.
5-5.3 Use equivalencies to convert units of measure within the metric system: converting length in millimeters, centimeters, meters, and kilometers; converting liquid volume in milliliters, centiliters, liters, and kiloliters; and converting mass in milligrams, centigrams, grams, and kilograms.
5-5.4 Apply formulas to determine the perimeters and areas of triangles, rectangles, and parallelograms.
5-5.5 Apply strategies and formulas to determine the volume of rectangular prisms.
5-5.6 Apply procedures to determine the amount of elapsed time in hours, minutes, and seconds within a 24-hour period.
5-5.7 Understand the relationship between the Celsius and Fahrenheit temperature scales.
5-5.8 Recall equivalencies associated with length, liquid volume, and mass:
10 millimeters = 1 centimeter, 100 centimeters = 1 meter, 1000 meters = 1 kilometer;
10 milliliters = 1 centiliter, 100 centiliters = 1 liter, 1000 liters = 1 kiloliter; and
10 milligrams = 1 centigram, 100 centigrams = 1 gram, 1000 grams = 1 kilogram.

GRADE 5

Data Analysis and Probability

Standard 5-6: The student will demonstrate through the mathematical processes an understanding of investigation design, the effect of data-collection methods on a data set, the interpretation and application of the measures of central tendency, and the application of basic concepts of probability.

Indicators

5-6.1 Design a mathematical investigation to address a question.
5-6.2 Analyze how data-collection methods affect the nature of the data set.
5-6.3 Apply procedures to calculate the measures of central tendency (mean, median, and mode).
5-6.4 Interpret the meaning and application of the measures of central tendency.
5-6.5 Represent the probability of a single-stage event in words and fractions.
5-6.6 Conclude why the sum of the probabilities of the outcomes of an experiment must equal 1.

Grade 6

Overview

This overview provides only the highlights of the new learning that should take place at the sixth-grade level. The specific skills and subject matter that sixth graders should be taught in each of the five mathematical strands are set forth in the formal standards and indicators for these strands. To alert educators as to when the progression in learning should occur for students in this grade, specific language is used with certain indicators:

- An indicator beginning with the phrase "**Generate strategies**" addresses a concept that is being formally introduced for the first time, and students must therefore be given experiences that foster conceptual understanding.
- An indicator beginning with the phrase "**Apply an algorithm**," "**Apply a procedure**," "**Apply procedures**," or "**Apply formulas**" addresses a concept that has been introduced in a previous grade: students should already have the conceptual understanding, and the goal must now be fluency.
- An indicator beginning with the phrase "**Apply strategies and formulas**" or "**Apply strategies and procedures**" addresses a concept that is being formally introduced for the first time, yet the goal must nonetheless be that students progress to fluency.

Highlights of the new learning for grade-six students are

- understanding the concepts of percentages and integers,
- comparing rational numbers and percentages,
- applying an algorithm to add and subtract fractions,
- generating strategies to multiply and divide fractions and decimals,
- understanding the concepts of exponents and powers of ten,
- applying order of operations,
- using inverse operations to solve one-step equations,
- representing location of points in all four quadrants,
- constructing two-dimensional shapes with rotational symmetry,
- classifying shapes as similar,
- identifying pairs of angles that are complementary or supplementary,
- applying strategies and formulas to approximate circumference and area of a circle,
- applying strategies and procedures to estimate and determine perimeters and areas of irregular shapes,
- using proportions to determine unit rates,
- using a scale to determine distance, and
- applying procedures to calculate the probability of complementary events.

GRADE 6

Mathematical Processes

The mathematical processes provide the framework for teaching, learning, and assessing in mathematics at all grade levels. Instructional programs should be built around these processes.

Standard 6-1: The student will understand and utilize the mathematical processes of problem solving, reasoning and proof, communication, connections, and representation.

The indicators for this standard, which are appropriate for grades six though eight, are adapted from *Principles and Standards for School Mathematics* (NCTM 2000). Classroom application should be based on the standard and its indicators; the mathematical goals for the class; and the skills, needs, and understandings of the particular students.

Indicators

6-1.1 Generate and solve complex abstract problems that involve modeling physical, social, and/or mathematical phenomena.
6-1.2 Evaluate conjectures and pose follow-up questions to prove or disprove conjectures.
6-1.3 Use inductive and deductive reasoning to formulate mathematical arguments.
6-1.4 Understand equivalent symbolic expressions as distinct symbolic forms that represent the same relationship.
6-1.5 Generalize mathematical statements based on inductive and deductive reasoning.
6-1.6 Use correct and clearly written or spoken words, variables, and notations to communicate about significant mathematical tasks.
6-1.7 Generalize connections among a variety of representational forms and real-world situations.
6-1.8 Use standard and nonstandard representations to convey and support mathematical relationships.

GRADE 6

Number and Operations

Standard 6-2: The student will demonstrate through the mathematical processes an understanding of the concepts of whole-number percentages, integers, and ratio and rate; the addition and subtraction of fractions; accurate, efficient, and generalizable methods of multiplying and dividing fractions and decimals; and the use of exponential notation to represent whole numbers.

Indicators

6-2.1 Understand whole-number percentages through 100.
6-2.2 Understand integers.
6-2.3 Compare rational numbers and whole-number percentages through 100 by using the symbols $\leq$, $\geq$, $<$, $>$, and $=$.
6-2.4 Apply an algorithm to add and subtract fractions.
6-2.5 Generate strategies to multiply and divide fractions and decimals.
6-2.6 Understand the relationship between ratio/rate and multiplication/division.
6-2.7 Apply strategies and procedures to determine values of powers of 10, up to 10^6.
6-2.8 Represent the prime factorization of numbers by using exponents.
6-2.9 Represent whole numbers in exponential form.

GRADE 6

Algebra

Standard 6-3: The student will demonstrate through the mathematical processes an understanding of writing, interpreting, and using mathematical expressions, equations, and inequalities.

Indicators

6-3.1 Analyze numeric and algebraic patterns and pattern relationships.
6-3.2 Apply order of operations to simplify whole-number expressions.
6-3.3 Represent algebraic relationships with variables in expressions, simple equations, and simple inequalities.
6-3.4 Use the commutative, associative, and distributive properties to show that two expressions are equivalent.
6-3.5 Use inverse operations to solve one-step equations that have whole-number solutions and variables with whole-number coefficients.

GRADE 6

Geometry

Standard 6-4: The student will demonstrate through the mathematical processes an understanding of shape, location, and movement within a coordinate system; similarity, complementary, and supplementary angles; and the relationship between line and rotational symmetry.

Indicators

6-4.1 Represent with ordered pairs of integers the location of points in a coordinate grid.
6-4.2 Apply strategies and procedures to find the coordinates of the missing vertex of a square, rectangle, or right triangle when given the coordinates of the polygon's other vertices.
6-4.3 Generalize the relationship between line symmetry and rotational symmetry for two-dimensional shapes.
6-4.4 Construct two-dimensional shapes with line or rotational symmetry.
6-4.5 Identify the transformation(s) used to move a polygon from one location to another in the coordinate plane.
6-4.6 Explain how transformations affect the location of the original polygon in the coordinate plane.
6-4.7 Compare the angles, side lengths, and perimeters of similar shapes.
6-4.8 Classify shapes as similar.
6-4.9 Classify pairs of angles as either complementary or supplementary.

GRADE 6

Measurement

Standard 6-5: The student will demonstrate through the mathematical processes an understanding of surface area; the perimeter and area of irregular shapes; the relationships among the circumference, diameter, and radius of a circle; the use of proportions to determine unit rates; and the use of scale to determine distance.

Indicators

6-5.1 Explain the relationships among the circumference, diameter, and radius of a circle.

6-5.2 Apply strategies and formulas with an approximation of *pi* (3.14, or $\frac{22}{7}$) to find the circumference and area of a circle.

6-5.3 Generate strategies to determine the surface area of a rectangular prism and a cylinder.

6-5.4 Apply strategies and procedures to estimate the perimeters and areas of irregular shapes.

6-5.5 Apply strategies and procedures of combining and subdividing to find the perimeters and areas of irregular shapes.

6-5.6 Use proportions to determine unit rates.

6-5.7 Use a scale to determine distance.

GRADE 6

Data Analysis and Probability

Standard 6-6: The student will demonstrate through the mathematical processes an understanding of the relationships within one population or sample.

Indicators

6-6.1 Predict the characteristics of one population based on the analysis of sample data.
6-6.2 Organize data in frequency tables, histograms, or stem-and-leaf plots as appropriate.
6-6.3 Analyze which measure of central tendency (mean, median, or mode) is the most appropriate for a given purpose.
6-6.4 Use theoretical probability to determine the sample space and probability for one- and two-stage events such as tree diagrams, models, lists, charts, and pictures.
6-6.5 Apply procedures to calculate the probability of complementary events.

Grade 7

Overview

This overview provides only the highlights of the new learning that should take place at the seventh-grade level. The specific skills and subject matter that seventh graders should be taught in each of the five mathematical strands are set forth in the formal standards and indicators for these strands. To alert educators as to when the progression in learning should occur for students in this grade, specific language is used with certain indicators:

- An indicator beginning with the phrase "**Generate strategies**" addresses a concept that is being formally introduced for the first time, and students must therefore be given experiences that foster conceptual understanding.
- An indicator beginning with the phrase "**Apply an algorithm**," "**Apply a procedure**," **"Apply procedures**," or "**Apply formulas**" addresses a concept that has been introduced in a previous grade: students should already have the conceptual understanding, and the goal must now be fluency.
- An indicator beginning with the phrase "**Apply strategies and formulas**" or "**Apply strategies and procedures**" addresses a concept that is being formally introduced for the first time, yet the goal must nonetheless be that students progress to fluency.

Highlights of the new learning for grade-seven students are

- understanding fractional percentages and percentages greater than 100;
- understanding the concept of square roots and the inverse relationship between squaring and finding square roots of perfect squares;
- understanding the meaning of absolute value;
- generating strategies to add, subtract, multiply, and divide integers;
- applying an algorithm to multiply and divide fractions and decimals;
- using inverse operations to solve two-step equations and inequalities;
- classifying and explaining proportional relationships;
- translating between two- and three-dimensional representations of compound figures;
- creating tessellations with transformations and explaining the angle-measure relationships among shapes that tessellate;
- applying strategies and formulas to determine the surface area and volume of three-dimensional shapes;
- using one-step unit analysis to convert between and within U.S. Customary System and the metric system; and
- applying procedures to calculate the interquartile range and the probability of mutually exclusive events.

GRADE 7

Mathematical Processes

The mathematical processes provide the framework for teaching, learning, and assessing in mathematics at all grade levels. Instructional programs should be built around these processes.

Standard 7-1: The student will understand and utilize the mathematical processes of problem solving, reasoning and proof, communication, connections, and representation.

The indicators for this standard, which are appropriate for grades six though eight, are adapted from *Principles and Standards for School Mathematics* (NCTM 2000). Classroom application should be based on the standard and its indicators; the mathematical goals for the class; and the skills, needs, and understandings of the particular students.

Indicators

7-1.1 Generate and solve complex abstract problems that involve modeling physical, social, or mathematical phenomena.
7-1.2 Evaluate conjectures and pose follow-up questions to prove or disprove conjectures.
7-1.3 Use inductive and deductive reasoning to formulate mathematical arguments.
7-1.4 Understand equivalent symbolic expressions as distinct symbolic forms that represent the same relationship.
7-1.5 Generalize mathematical statements based on inductive and deductive reasoning.
7-1.6 Use correct and clearly written or spoken words, variables, and notation to communicate about significant mathematical tasks.
7-1.7 Generalize connections among a variety of representational forms and real-world situations.
7-1.8 Use standard and nonstandard representations to convey and support mathematical relationships.

GRADE 7

Number and Operations

Standard 7-2: The student will demonstrate through the mathematical processes an understanding of the representation of rational numbers, percentages, and square roots of perfect squares; the application of ratios, rates, and proportions to solve problems; accurate, efficient, and generalizable methods for operations with integers; the multiplication and division of fractions and decimals; and the inverse relationship between squaring and finding the square roots of perfect squares.

Indicators

7-2.1 Understand fractional percentages and percentages greater than one hundred.
7-2.2 Represent the location of rational numbers and square roots of perfect squares on a number line.
7-2.3 Compare rational numbers, percentages, and square roots of perfect squares by using the symbols $\leq$, $\geq$, $<$, $>$, and $=$.
7-2.4 Understand the meaning of absolute value.
7-2.5 Apply ratios, rates, and proportions to discounts, taxes, tips, interest, unit costs, and similar shapes.
7-2.6 Translate between standard form and exponential form.
7-2.7 Translate between standard form and scientific notation.
7-2.8 Generate strategies to add, subtract, multiply, and divide integers.
7-2.9 Apply an algorithm to multiply and divide fractions and decimals.
7-2.10 Understand the inverse relationship between squaring and finding the square roots of perfect squares.

GRADE 7

Algebra

Standard 7-3: The student will demonstrate through the mathematical processes an understanding of proportional relationships.

Indicators

7-3.1 Analyze geometric patterns and pattern relationships.
7-3.2 Analyze tables and graphs to describe the rate of change between and among quantities.
7-3.3 Understand slope as a constant rate of change.
7-3.4 Use inverse operations to solve two-step equations and two-step inequalities.
7-3.5 Represent on a number line the solution of a two-step inequality.
7-3.6 Represent proportional relationships with graphs, tables, and equations.
7-3.7 Classify relationships as either directly proportional, inversely proportional, or nonproportional.

GRADE 7

Geometry

Standard 7-4: The student will demonstrate through the mathematical processes an understanding of proportional reasoning, tessellations, the use of geometric properties to make deductive arguments. the results of the intersection of geometric shapes in a plane, and the relationships among angles formed when a transversal intersects two parallel lines.

Indicators

7-4.1 Analyze geometric properties and the relationships among the properties of triangles, congruence, similarity, and transformations to make deductive arguments.
7-4.2 Explain the results of the intersection of two or more geometric shapes in a plane.
7-4.3 Illustrate the cross section of a solid.
7-4.4 Translate between two- and three-dimensional representations of compound figures.
7-4.5 Analyze the congruent and supplementary relationships—specifically, alternate interior, alternate exterior, corresponding, and adjacent—of the angles formed by parallel lines and a transversal.
7-4.6 Compare the areas of similar shapes and the areas of congruent shapes.
7-4.7 Explain the proportional relationship among attributes of similar shapes.
7-4.8 Apply proportional reasoning to find missing attributes of similar shapes.
7-4.9 Create tessellations with transformations.
7-4.10 Explain the relationship of the angle measurements among shapes that tessellate.

GRADE 7

Measurement

Standard 7-5: The student will demonstrate through the mathematical processes an understanding of how to use ratio and proportion to solve problems involving scale factors and rates and how to use one-step unit analysis to convert between and within the U.S. Customary System and the metric system.

Indicators

7-5.1 Use ratio and proportion to solve problems involving scale factors and rates.

7-5.2 Apply strategies and formulas to determine the surface area and volume of the three-dimensional shapes prism, pyramid, and cylinder.

7-5.3 Generate strategies to determine the perimeters and areas of trapezoids.

7-5.4 Recall equivalencies associated with length, mass and weight, and liquid volume: 1 square yard = 9 square feet, 1 cubic meter = 1 million cubic centimeters, 1 kilometer = $\frac{5}{8}$ mile, 1 inch = 2.54 centimeters; 1 kilogram = 2.2 pounds; and 1.06 quarts = 1 liter.

7-5.5 Use one-step unit analysis to convert between and within the U.S. Customary System and the metric system.

GRADE 7

Data Analysis and Probability

Standard 7-6: The student will demonstrate through the mathematical processes an understanding of the relationships between two populations or samples.

Indicators

7-6.1 Predict the characteristics of two populations based on the analysis of sample data.
7-6.2 Organize data in box plots or circle graphs as appropriate.
7-6.3 Apply procedures to calculate the interquartile range.
7-6.4 Interpret the interquartile range for data.
7-6.5 Apply procedures to calculate the probability of mutually exclusive simple or compound events.
7-6.6 Interpret the probability of mutually exclusive simple or compound events.
7-6.7 Differentiate between experimental and theoretical probability of the same event.
7-6.8 Use the fundamental counting principle to determine the number of possible outcomes for a multistage event.

Grade 8

Overview

This overview provides only the highlights of the new learning that should take place at the seventh-grade level. The specific skills and subject matter that seventh graders should be taught in each of the five mathematical strands are set forth in the formal standards and indicators for these strands. To alert educators as to when the progression in learning should occur for students in this grade, specific language is used with certain indicators:

- An indicator beginning with the phrase "**Generate strategies**" addresses a concept that is being formally introduced for the first time, and students must therefore be given experiences that foster conceptual understanding.
- An indicator beginning with the phrase "**Apply an algorithm**," "**Apply a procedure**," **"Apply procedures**," or "**Apply formulas**" addresses a concept that has been introduced in a previous grade: students should already have the conceptual understanding, and the goal must now be fluency.
- An indicator beginning with the phrase "**Apply strategies and formulas**" or "**Apply strategies and procedures**" addresses a concept that is being formally introduced for the first time, yet the goal must nonetheless be that students progress to fluency.

Highlights of the new learning for grade-eight students are

- applying an algorithm to add, subtract, multiply, and divide integers;
- understanding the concept of irrational numbers;
- applying procedures to approximate square and cube roots;
- applying procedures to solve multistep equations;
- classifying relationships between two variables as either linear or nonlinear;
- identifying the coordinates of the *x*- and *y*-intercepts of a linear equation;
- understanding slope as a constant rate of change;
- applying the Pythagorean theorem;
- using ordered pairs, equations, intercepts, and intersections to locate points and lines in a coordinate plane;
- applying a dilation on a square, rectangle, or right triangle in a coordinate plane and analyzing the effect;
- applying strategies and formulas to determine volume of three-dimensional shapes;
- using multistep unit analysis to convert between and with the U.S. Customary System and the metric system; and
- applying procedures to compute the odds of a given event.

GRADE 8

Mathematical Processes

The mathematical processes provide the framework for teaching, learning, and assessing in mathematics at all grade levels. Instructional programs should be built around these processes.

Standard 8-1: The student will understand and utilize the mathematical processes of problem solving, reasoning and proof, communication, connections, and representation.

The indicators for this standard, which are appropriate for grades six though eight, are adapted from *Principles and Standards for School Mathematics* (NCTM 2000). Classroom application should be based on the standard and its indicators; the mathematical goals for the class; and the skills, needs, and understandings of the particular students.

Indicators

8-1.1 Generate and solve complex abstract problems that involve modeling physical, social, or mathematical phenomena.
8-1.2 Evaluate conjectures and pose follow-up questions to prove or disprove conjectures.
8-1.3 Use inductive and deductive reasoning to formulate mathematical arguments.
8-1.4 Understand equivalent symbolic expressions as distinct symbolic forms that represent the same relationship.
8-1.5 Generalize mathematical statements based on inductive and deductive reasoning.
8-1.6 Use correct and clearly written or spoken words, variables, and notations to communicate about significant mathematical tasks.
8-1.7 Generalize connections among a variety of representational forms and real-world situations.
8-1.8 Use standard and nonstandard representations to convey and support mathematical relationships.

GRADE 8

Number and Operations

Standard 8-2: The student will demonstrate through the mathematical processes an understanding of operations with integers, the effects of multiplying and dividing with rational numbers, the comparative magnitude of rational and irrational numbers, the approximation of cube and square roots, and the application of proportional reasoning.

Indicators

8-2.1 Apply an algorithm to add, subtract, multiply, and divide integers.
8-2.2 Understand the effect of multiplying and dividing a rational number by another rational number.
8-2.3 Represent the approximate location of irrational numbers on a number line.
8-2.4 Compare rational and irrational numbers by using the symbols $\leq$, $\geq$, $<$, $>$, and $=$.
8-2.5 Apply the concept of absolute value.
8-2.6 Apply strategies and procedures to approximate between two whole numbers the square roots or cube roots of numbers less than 1,000.
8-2.7 Apply ratios, rates, and proportions.

GRADE 8

Algebra

Standard 8-3: The student will demonstrate through the mathematical processes an understanding of equations, inequalities, and linear functions.

Indicators

8-3.1 Translate among verbal, graphic, tabular, and algebraic representations of linear functions.
8-3.2 Represent algebraic relationships with equations and inequalities.
8-3.3 Use commutative, associative, and distributive properties to examine the equivalence of a variety of algebraic expressions.
8-3.4 Apply procedures to solve multistep equations.
8-3.5 Classify relationships between two variables in graphs, tables, and/or equations as either linear or nonlinear.
8-3.6 Identify the coordinates of the *x*- and *y*-intercepts of a linear equation from a graph, equation, and/or table.
8-3.7 Identify the slope of a linear equation from a graph, equation, and/or table.

GRADE 8

Geometry

Standard 8-4: The student will demonstrate through the mathematical processes an understanding of the Pythagorean theorem; the use of ordered pairs, equations, intercepts, and intersections to locate points and lines in a coordinate plane; and the effect of a dilation in a coordinate plane.

Indicators

8-4.1 Apply the Pythagorean theorem.
8-4.2 Use ordered pairs, equations, intercepts, and intersections to locate points and lines in a coordinate plane.
8-4.3 Apply a dilation to a square, rectangle, or right triangle in a coordinate plane.
8-4.4 Analyze the effect of a dilation on a square, rectangle, or right triangle in a coordinate plane.

GRADE 8

Measurement

Standard 8-5: The student will demonstrate through the mathematical processes an understanding of the proportionality of similar figures; the necessary levels of accuracy and precision in measurement; the use of formulas to determine circumference, perimeter, area, and volume; and the use of conversions within and between the U.S. Customary System and the metric system..

Indicators

8-5.1 Use proportional reasoning and the properties of similar shapes to determine the length of a missing side.

8-5.2 Explain the effect on the area of two-dimensional shapes and on the volume of three-dimensional shapes when one or more of the dimensions are changed.

8-5.3 Apply strategies and formulas to determine the volume of the three-dimensional shapes cone and sphere.

8-5.4 Apply formulas to determine the exact (*pi)* circumference and area of a circle.

8-5.5 Apply formulas to determine the perimeters and areas of trapezoids.

8-5.6 Analyze a variety of measurement situations to determine the necessary level of accuracy and precision.

8-5.7 Use multistep unit analysis to convert between and within U.S. Customary System and the metric system.

GRADE 8

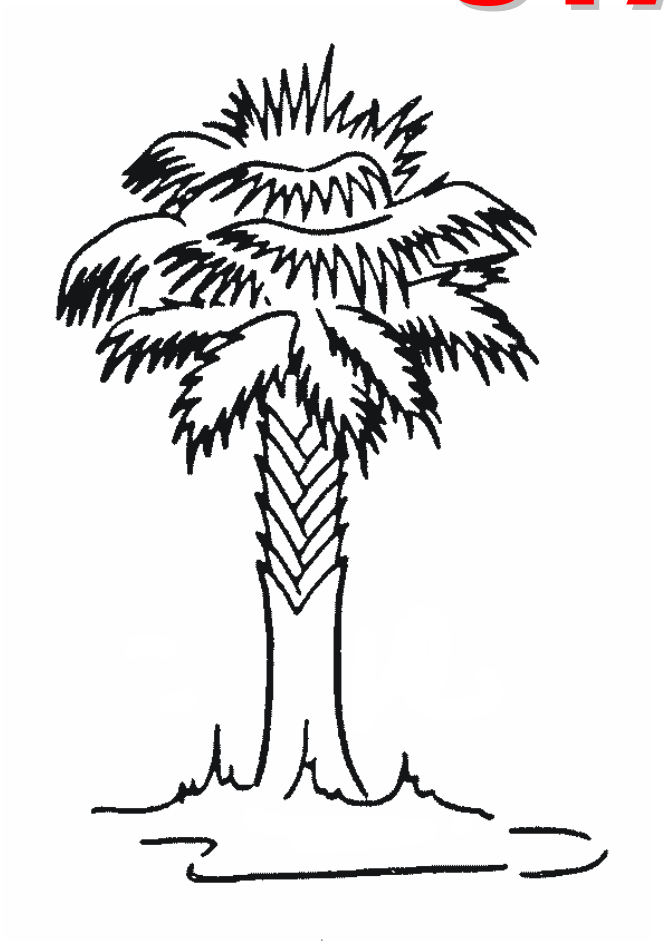
Data Analysis and Probability

Standard 8-6: The student will demonstrate through the mathematical processes an understanding of the relationships between two variables within one population or sample.

Indicators

8-6.1 Generalize the relationship between two sets of data by using scatterplots and lines of best fit.
8-6.2 Organize data in matrices or scatterplots as appropriate.
8-6.3 Use theoretical and experimental probability to make inferences and convincing arguments about an event or events.
8-6.4 Apply procedures to calculate the probability of two dependent events.
8-6.5 Interpret the probability for two dependent events.
8-6.6 Apply procedures to compute the odds of a given event.
8-6.7 Analyze probability using area models.
8-6.8 Interpret graphic and tabular data representations by using range and the measures of central tendency (mean, median, and mode).

HIGH SCHOOL CORE-AREA STANDARDS



Elementary Algebra

Overview

The academic standards for the elementary algebra core area establish the process skills and core content for Algebra 1, Mathematics for the Technologies 1, and Mathematics for the Technologies 2, which should provide students with the mathematics skills and conceptual understanding necessary for them to further their mathematical education or to pursue mathematics-related technical careers. These standards will be the basis for the development of the items on the state-required end-of-course examination for Algebra 1 and Mathematics for the Technologies 2.

The content of the elementary algebra standards encompasses the real number system; operations involving exponents, matrices, and algebraic expressions; relations and functions; writing and solving linear equations; graphs and characteristics of linear equations; and quadratic relationships and functions. Teachers, schools, and districts should use the elementary algebra standards to make decisions concerning the structure and content of Algebra 1, Mathematics for the Technologies 1, and Mathematics for the Technologies 2. Content in these three courses may go beyond the elementary algebra standards.

All courses based on the academic standards for elementary algebra must include instruction using the mathematics process standards, allowing students to engage in problem solving, decision making, critical thinking, and applied learning. Educators must determine the extent to which such courses or individual classes may go beyond these standards. Such decisions will involve choices regarding additional content, activities, and learning strategies and will depend on the objectives of the particular courses or individual classes.

In all courses based on the elementary algebra standards, hand-held graphing calculators are required for instruction and assessment. Students should learn to use a variety of ways to represent data, to use a variety of mathematical tools such as graph paper, and to use technologies such as graphing calculators to solve problems.

Note: The term *including* appears in parenthetical statements in the high school mathematics indicators to introduce a list of specifics that are intended to clarify and focus the teaching and learning of the particular concept. That is, within these parenthetical including statements are specified the components of the indicator that are critical for the particular core area with regard both to the state assessments and to the management of time in the classroom. While instruction must focus on the entire indicator, educators must be certain to cover the components specified in the parenthetical *including* statements.

HIGH SCHOOL CORE AREA

Elementary Algebra

The mathematical processes provide the framework for teaching, learning, and assessing in all high school mathematics core courses. Instructional programs should be built around these processes.

Standard EA-1: The student will understand and utilize the mathematical processes of problem solving, reasoning and proof, communication, connections, and representation.

Indicators

EA-1.1 Communicate a knowledge of algebraic relationships by using mathematical terminology appropriately.
EA-1.2 Connect algebra with other branches of mathematics.
EA-1.3 Apply algebraic methods to solve problems in real-world contexts.
EA-1.4 Judge the reasonableness of mathematical solutions.
EA-1.5 Demonstrate an understanding of algebraic relationships by using a variety of representations (including verbal, graphic, numerical, and symbolic).
EA-1.6 Understand how algebraic relationships can be represented in concrete models, pictorial models, and diagrams.
EA-1.7 Understand how to represent algebraic relationships by using tools such as handheld computing devices, spreadsheets, and computer algebra systems (CASs).

HIGH SCHOOL CORE AREA

Elementary Algebra

Standard EA-2: The student will demonstrate through the mathematical processes an understanding of the real number system and operations involving exponents, matrices, and algebraic expressions.

Indicators

EA-2.1 Exemplify elements of the real number system (including integers, rational numbers, and irrational numbers).

EA-2.2 Apply the laws of exponents and roots to solve problems.

EA-2.3 Carry out a procedure to perform operations (including multiplication and division) with numbers written in scientific notation.

EA-2.4 Use dimensional analysis to convert units of measure within a system.

EA-2.5 Carry out a procedure using the properties of real numbers (including commutative, associative, and distributive) to simplify expressions.

EA-2.6 Carry out a procedure to evaluate an expression by substituting a value for the variable.

EA-2.7 Carry out a procedure (including addition, subtraction, multiplication, and division by a monomial) to simplify polynomial expressions.

EA-2.8 Carry out a procedure to factor binomials, trinomials, and polynomials by using various techniques (including the greatest common factor, the difference between two squares, and quadratic trinomials).

EA-2.9 Carry out a procedure to perform operations with matrices (including addition, subtraction, and scalar multiplication).

EA-2.10 Represent applied problems by using matrices.

HIGH SCHOOL CORE AREA

Elementary Algebra

Standard EA-3: The student will demonstrate through the mathematical processes an understanding of relationships and functions.

Indicators

EA-3.1 Classify a relationship as being either a function or not a function when given data as a table, set of ordered pairs, or graph.

EA-3.2 Use function notation to represent functional relationships.

EA-3.3 Carry out a procedure to evaluate a function for a given element in the domain.

EA-3.4 Analyze the graph of a continuous function to determine the domain and range of the function.

EA-3.5 Carry out a procedure to graph parent functions (including $y = x$, $y = x^2$, $y = \sqrt{x}$, $y = |x|$, and $y = \frac{1}{x}$).

EA-3.6 Classify a variation as either direct or inverse.

EA-3.7 Carry out a procedure to solve literal equations for a specified variable.

EA-3.8 Apply proportional reasoning to solve problems.

HIGH SCHOOL CORE AREA

Elementary Algebra

Standard EA-4: The student will demonstrate through the mathematical processes an understanding of the procedures for writing and solving linear equations and inequalities.

Indicators

EA-4.1 Carry out a procedure to write an equation of a line with a given slope and a *y*-intercept.
EA-4.2 Carry out a procedure to write an equation of a line with a given slope passing through a given point.
EA-4.3 Carry out a procedure to write an equation of a line passing through two given points.
EA-4.4 Use a procedure to write an equation of a trend line from a given scatterplot.
EA-4.5 Analyze a scatterplot to make predictions.
EA-4.6 Represent linear equations in multiple forms (including point-slope, slope-intercept, and standard).
EA-4.7 Carry out procedures to solve linear equations for one variable algebraically.
EA-4.8 Carry out procedures to solve linear inequalities for one variable algebraically and then to graph the solution.
EA-4.9 Carry out a procedure to solve systems of two linear equations graphically.
EA-4.10 Carry out a procedure to solve systems of two linear equations algebraically.

HIGH SCHOOL CORE AREA

Elementary Algebra

Standard EA-5: The student will demonstrate through the mathematical processes an understanding of the graphs and characteristics of linear equations and inequalities.

Indicators

EA-5.1 Carry out a procedure to graph a line when given the equation of the line.

EA-5.2 Analyze the effects of changes in the slope, m, and the y-intercept, b, on the graph of $y = mx + b$.

EA-5.3 Carry out a procedure to graph the line with a given slope and a y-intercept.

EA-5.4 Carry out a procedure to graph the line with a given slope passing through a given point.

EA-5.5 Carry out a procedure to determine the x-intercept and y-intercept of lines from data given tabularly, graphically, symbolically, and verbally.

EA-5.6 Carry out a procedure to determine the slope of a line from data given tabularly, graphically, symbolically, and verbally.

EA-5.7 Apply the concept of slope as a rate of change to solve problems.

EA-5.8 Analyze the equations of two lines to determine whether the lines are perpendicular or parallel.

EA-5.9 Analyze given information to write a linear function that models a given problem situation.

EA-5.10 Analyze given information to determine the domain and range of a linear function in a problem situation.

EA-5.11 Analyze given information to write a system of linear equations that models a given problem situation.

EA-5.12 Analyze given information to write a linear inequality in one variable that models a given problem situation.

HIGH SCHOOL CORE AREA

Elementary Algebra

Standard EA-6: The student will demonstrate through the mathematical processes an understanding of quadratic relationships and functions.

Indicators

EA-6.1 Analyze the effects of changing the leading coefficient *a* on the graph of $y = ax^2$.

EA-6.2 Analyze the effects of changing the constant *c* on the graph of $y = x^2 + c$.

EA-6.3 Analyze the graph of a quadratic function to determine its equation.

EA-6.4 Carry out a procedure to solve quadratic equations by factoring.

EA-6.5 Carry out a graphic procedure to approximate the solutions of quadratic equations.

EA-6.6 Analyze given information to determine the domain of a quadratic function in a problem situation.

Intermediate Algebra

Overview

The academic standards for the intermediate algebra core area establish the process skills and core content for Algebra 2, which should provide students with the mathematics skills and conceptual understanding necessary for them to further their mathematical education or to pursue mathematics-related technical careers.

The content of the intermediate algebra standards encompasses functions; systems of equations; systems of linear inequalities; quadratic equations; complex numbers; algebraic expressions; nonlinear relationships including exponential, logarithmic, radical, polynomial, and rational; conic sections; and sequences and series. Teachers, schools, and districts should use the intermediate algebra standards to make decisions concerning the structure and content of Algebra 2. Content in this course may go beyond the intermediate algebra standards.

All courses based on the academic standards for intermediate algebra must include instruction using the mathematics process standards, allowing students to engage in problem solving, decision making, critical thinking, and applied learning. Educators must determine the extent to which such courses or individual classes may go beyond these standards. Such decisions will involve choices regarding additional content, activities, and learning strategies and will depend on the objectives of the particular courses or individual classes.

In all courses based on the intermediate algebra standards, hand-held graphing calculators are required for instruction and assessment. Students should learn to use a variety of ways to represent data, to use a variety of mathematical tools such as graph paper, and to use technologies such as graphing calculators to solve problems.

Note: The term *including* appears in parenthetical statements in the high school mathematics indicators to introduce a list of specifics that are intended to clarify and focus the teaching and learning of the particular concept. That is, within these parenthetical including statements are specified the components of the indicator that are critical for the particular core area with regard both to the state assessments and to the management of time in the classroom. While instruction must focus on the entire indicator, educators must be certain to cover the components specified in the parenthetical *including* statements.

HIGH SCHOOL CORE AREA

Intermediate Algebra

The mathematical processes provide the framework for teaching, learning, and assessing in all high school mathematics core courses. Instructional programs should be built around these processes.

Standard IA-1: The student will understand and utilize the mathematical processes of problem solving, reasoning and proof, communication, connections, and representation.

Indicators

IA-1.1 Communicate a knowledge of algebraic relationships by using mathematical terminology appropriately.
IA-1.2 Connect algebra with other branches of mathematics.
IA-1.3 Apply algebraic methods to solve problems in real-world contexts.
IA-1.4 Judge the reasonableness of mathematical solutions.
IA-1.5 Demonstrate an understanding of algebraic relationships by using a variety of representations (including verbal, graphic, numerical, and symbolic).
IA-1.6 Understand how algebraic relationships can be represented in concrete models, pictorial models, and diagrams.
IA-1.7 Understand how to represent algebraic relationships by using tools such as handheld computing devices, spreadsheets, and computer algebra systems (CASs).

HIGH SCHOOL CORE AREA

Intermediate Algebra

Standard IA-2: The student will demonstrate through the mathematical processes an understanding of functions, systems of equations, and systems of linear inequalities.

Indicators

IA-2.1 Carry out a procedure to solve a system of linear inequalities algebraically.

IA-2.2 Carry out a procedure to solve a system of linear inequalities graphically.

IA-2.3 Analyze a problem situation to determine a system of linear inequalities that models the problem situation.

IA-2.4 Use linear programming to solve contextual problems involving a system of linear inequalities.

IA-2.5 Carry out procedures to perform operations on polynomial functions (including $f(x) + g(x)$, $f(x) - g(x)$, $f(x) \bullet g(x)$, and $f(x)/g(x)$).

IA-2.6 Apply a procedure to write the equation of a composition of given functions.

IA-2.7 Carry out a procedure to graph translations of parent functions (including $y = x$, $y = x^2$, $y = \sqrt{x}$, $y = |x|$, and $y = \frac{1}{x}$).

IA-2.8 Carry out a procedure to graph transformations of parent functions (including $y = x$, $y = x^2$, and $y = |x|$).

IA-2.9 Carry out a procedure to graph discontinuous functions (including piecewise and step functions).

IA-2.10 Carry out a procedure to determine the domain and range of discontinuous functions (including piecewise and step functions).

IA-2.11 Carry out a procedure to solve a system of equations (including two linear functions and one linear function with one quadratic function).

HIGH SCHOOL CORE AREA

Intermediate Algebra

Standard IA-3: The student will demonstrate through the mathematical processes an understanding of quadratic equations and the complex number system.

Indicators

IA-3.1 Carry out a procedure to simplify expressions involving powers of *i*.
IA-3.2 Carry out a procedure to perform operations with complex numbers (including addition, subtraction, multiplication, and division).
IA-3.3 Carry out a procedure to solve quadratic equations algebraically (including factoring, completing the square, and applying the quadratic formula).
IA-3.4 Use the discriminant to determine the number and type of solutions of a quadratic equation.
IA-3.5 Analyze given information (including quadratic models) to solve contextual problems.
IA-3.6 Carry out a procedure to write an equation of a quadratic function when given its roots.

HIGH SCHOOL CORE AREA

Intermediate Algebra

Standard IA-4: The student will demonstrate through the mathematical processes an understanding of algebraic expressions and nonlinear functions.

Indicators

IA-4.1 Carry out a procedure to perform operations (including multiplication, exponentiation, and division) with polynomial expressions.

IA-4.2 Carry out a procedure to determine specified points (including zeros, maximums, and minimums) of polynomial functions.

IA-4.3 Carry out a procedure to solve polynomial equations (including factoring by grouping, factoring the difference between two squares, factoring the sum of two cubes, and factoring the difference between two cubes).

IA-4.4 Analyze given information (including polynomial models) to solve contextual problems.

IA-4.5 Carry out a procedure to simplify algebraic expressions involving rational exponents.

IA-4.6 Carry out a procedure to simplify algebraic expressions involving logarithms.

IA-4.7 Carry out a procedure to perform operations with expressions involving rational exponents (including addition, subtraction, multiplication, division, and exponentiation).

IA-4.8 Carry out a procedure to perform operations with rational expressions (including addition, subtraction, multiplication, and division).

IA-4.9 Carry out a procedure to solve radical equations algebraically.

IA-4.10 Carry out a procedure to solve logarithmic equations algebraically.

IA-4.11 Carry out a procedure to solve logarithmic equations graphically.

IA-4.12 Carry out a procedure to solve rational equations algebraically.

IA-4.13 Carry out a procedure to graph logarithmic functions.

IA-4.14 Carry out a procedure to graph exponential functions.

HIGH SCHOOL CORE AREA

Intermediate Algebra

Standard IA-5: The student will demonstrate through the mathematical processes an understanding of conic sections.

Indicators

IA-5.1 Carry out a procedure to graph the circle whose equation is the form $x^2 + y^2 = r^2$.

IA-5.2 Carry out a procedure to write an equation of a circle centered at the origin when given its radius.

IA-5.3 Carry out a procedure to graph the ellipse whose equation is the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

IA-5.4 Carry out a procedure to write an equation of an ellipse centered at the origin when given information from among length of major axis, length of minor axis, and vertices.

IA-5.5 Carry out a procedure to graph the hyperbola whose equation is the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

IA-5.6 Carry out a procedure to write an equation of a hyperbola centered at the origin with specified vertices.

IA-5.7 Match the equation of a conic section with its graph.

HIGH SCHOOL CORE AREA

Intermediate Algebra

Standard IA-6: The student will demonstrate through the mathematical processes an understanding of sequences and series.

Indicators

IA-6.1 Categorize a sequence as arithmetic, geometric, or neither.

IA-6.2 Carry out a procedure to write a specified term of an arithmetic or geometric sequence when given the nth term of the sequence.

IA-6.3 Carry out a procedure to write a formula for the nth term of an arithmetic or geometric sequence when given at least four consecutive terms of the sequence.

IA-6.4 Carry out a procedure to write a formula for the nth term of an arithmetic or geometric sequence when given at least four terms of the sequence.

IA-6.5 Represent an arithmetic or geometric series by using sigma notation.

IA-6.6 Carry out a procedure to calculate the sum of an arithmetic or geometric series written in sigma notation.

IA-6.7 Carry out a procedure to determine consecutive terms of a sequence that is defined recursively.

IA-6.8 Carry out a procedure to define a sequence recursively when given four or more consecutive terms of the sequence.

IA-6.9 Translate between the explicit form and the recursive form of sequences.

Geometry

Overview

The academic standards for the geometry core area establish the process skills and core content for Geometry and Mathematics for the Technologies 3, which should provide students with the mathematics skills and conceptual understanding necessary for them to further their mathematical education or to pursue mathematics-related technical careers.

The content of the geometry standards encompasses properties of basic geometric figures; properties of triangles; properties of quadrilaterals and other polygons; properties of circles, lines, and special segments intersecting circles; transformations; coordinate geometry; vectors; surface area and volume of three-dimensional objects; and proofs. Teachers, schools, and districts should use the geometry standards to make decisions concerning the structure and content of Geometry and Mathematics for the Technologies 3. Content in these two courses may go beyond the geometry standards.

All courses based on the academic standards for geometry must include instruction using the mathematics process standards, allowing students to engage in problem solving, decision making, critical thinking, and applied learning. Educators must determine the extent to which such courses or individual classes may go beyond these standards. Such decisions will involve choices regarding additional content, activities, and learning strategies and will depend on the objectives of the particular courses or individual classes.

In all courses based on the geometry standards, appropriate tools and technologies are required for instruction and assessment. Students should learn to use a variety of ways to represent data, to use a variety of mathematical tools such as graph paper, and to use technologies such as graphing calculators to solve problems.

Note: The term *including* appears in parenthetical statements in the high school mathematics indicators to introduce a list of specifics that are intended to clarify and focus the teaching and learning of the particular concept. That is, within these parenthetical including statements are specified the components of the indicator that are critical for the particular core area with regard both to the state assessments and to the management of time in the classroom. While instruction must focus on the entire indicator, educators must be certain to cover the components specified in the parenthetical *including* statements.

HIGH SCHOOL CORE AREA

Geometry

The mathematical processes provide the framework for teaching, learning, and assessing in all high school mathematics core courses. Instructional programs should be built around these processes.

Standard G-1: The student will understand and utilize the mathematical processes of problem solving, reasoning and proof, communication, connections, and representation.

Indicators

G-1.1 Demonstrate an understanding of the axiomatic structure of geometry by using undefined terms, definitions, postulates, theorems, and corollaries.

G-1.2 Communicate knowledge of geometric relationships by using mathematical terminology appropriately.

G-1.3 Apply basic rules of logic to determine the validity of the converse, inverse, and contrapositive of a conditional statement.

G-1.4 Formulate and test conjectures by using a variety of tools such as concrete models, graphing calculators, spreadsheets, and dynamic geometry software.

G-1.5 Use inductive reasoning to formulate conjectures.

G-1.6 Use deductive reasoning to validate conjectures with formal and informal proofs, and give counterexamples to disprove a statement.

G-1.7 Understand the historical development of geometry.

G-1.8 Connect geometry with other branches of mathematics.

G-1.9 Demonstrate an understanding of how geometry applies to in real-world contexts (including architecture, construction, farming, and astronomy).

G-1.10 Demonstrate an understanding of geometric relationships (including constructions through investigations by using a variety of tools such as straightedge, compass, Patty Paper, dynamic geometry software, and handheld computing devices).

HIGH SCHOOL CORE AREA

Geometry

Standard G-2: The student will demonstrate through the mathematical processes an understanding of the properties of basic geometric figures and the relationships between and among them.

Indicators

G-2.1 Infer missing elements of visual or numerical geometric patterns (including triangular and rectangular numbers and the number of diagonals in polygons).

G-2.2 Apply properties of parallel lines, intersecting lines, and parallel lines cut by a transversal to solve problems.

G-2.3 Use the congruence of line segments and angles to solve problems.

G-2.4 Use direct measurement to determine the length of a segment, degree of an angle, and distance from a point to a line.

G-2.5 Carry out a procedure to create geometric constructions (including the midpoint of a line segment, the angle bisector, the perpendicular bisector of a line segment, the line through a given point that is parallel to a given line, and the line through a given point that is perpendicular to a given line).

G-2.6 Use scale factors to solve problems involving scale drawings and models.

G-2.7 Use geometric probability to solve problems.

HIGH SCHOOL CORE AREA

Geometry

Standard G-3: The student will demonstrate through the mathematical processes an understanding of the properties and special segments of triangles and the relationships between and among triangles.

Indicators

G-3.1 Carry out a procedure to compute the perimeter of a triangle.
G-3.2 Carry out a procedure to compute the area of a triangle.
G-3.3 Analyze how changes in dimensions affect the perimeter or area of triangles.
G-3.4 Apply properties of isosceles and equilateral triangles to solve problems.
G-3.5 Use interior angles, exterior angles, medians, angle bisectors, altitudes, and perpendicular bisectors to solve problems.
G-3.6 Apply the triangle sum theorem to solve problems.
G-3.7 Apply the triangle inequality theorem to solve problems.
G-3.8 Apply congruence and similarity relationships among triangles to solve problems.
G-3.9 Apply theorems to prove that triangles are either similar or congruent.
G-3.10 Use the Pythagorean theorem and its converse to solve problems.
G-3.11 Use the properties of 45-45-90 and 30-60-90 triangles to solve problems.
G-3.12 Use trigonometric ratios (including sine, cosine, and tangent) to solve problems involving right triangles.

HIGH SCHOOL CORE AREA

Geometry

Standard G-4: The student will demonstrate through the mathematical processes an understanding of the properties of quadrilaterals and other polygons and the relationships between and among them.

Indicators

G-4.1 Carry out a procedure to compute the perimeter of quadrilaterals, regular polygons, and composite figures.

G-4.2 Carry out a procedure to find the area of quadrilaterals, regular polygons, and composite figures.

G-4.3 Apply procedures to compute measures of interior and exterior angles of polygons.

G-4.4 Analyze how changes in dimensions affect the perimeter or area of quadrilaterals and regular polygons.

G-4.5 Apply the properties and attributes of quadrilaterals and regular polygons and their component parts to solve problems.

G-4.6 Apply congruence and similarity relationships among shapes (including quadrilaterals and polygons) to solve problems.

HIGH SCHOOL CORE AREA

Geometry

Standard G-5: The student will demonstrate through the mathematical processes an understanding of the properties of circles, the lines that intersect them, and the use of their special segments.

Indicators

G-5.1 Carry out a procedure to compute the circumference of circles.
G-5.2 Carry out a procedure to compute the area of circles.
G-5.3 Analyze how a change in the radius affects the circumference or area of a circle.
G-5.4 Carry out a procedure to compute the length of an arc or the area of a sector of a circle.
G-5.5 Apply the properties of the component parts of a circle (including radii, diameters, chords, sectors, arcs, and segments) to solve problems.
G-5.6 Apply the properties of lines that intersect circles (including two secants, two tangents, and a secant and a tangent) to solve problems.
G-5.7 Apply the properties of central angles, inscribed angles, and arcs of circles to solve problems.

HIGH SCHOOL CORE AREA

Geometry

Standard G-6: The student will demonstrate through the mathematical processes an understanding of transformations, coordinate geometry, and vectors.

Indicators

G-6.1 Use the distance formula to solve problems.
G-6.2 Use the midpoint formula to solve problems.
G-6.3 Apply transformations—translation, reflection, rotation, and dilation—to figures in the coordinate plane by using sketches and coordinates.
G-6.4 Apply transformations (including translation and dilation) to figures in the coordinate plane by using matrices.
G-6.5 Carry out a procedure to represent the sum of two vectors geometrically by using the parallelogram method.
G-6.6 Carry out a procedure to determine the magnitude and direction of the resultant of two vectors by using a scale drawing and direct measurement.
G-6.7 Carry out a procedure to compute the magnitude of the resultant of two perpendicular vectors by using the Pythagorean theorem.
G-6.8 Carry out a procedure to determine the direction of the resultant of two perpendicular vectors by using a scale drawing and direct measurement.

HIGH SCHOOL CORE AREA

Geometry

Standard G-7: The student will demonstrate through the mathematical processes an understanding of the surface area and volume of three-dimensional objects.

Indicators

G-7.1 Carry out a procedure to compute the surface area of three-dimensional objects (including cones, cylinders, pyramids, prisms, spheres, and hemispheres).

G-7.2 Carry out a procedure to compute the volume of three-dimensional objects (including cones, cylinders, pyramids, prisms, spheres, hemispheres, and composite objects).

G-7.3 Analyze how changes in dimensions affect the volume of objects (including cylinders, prisms, and spheres).

G-7.4 Apply congruence and similarity relationships among geometric objects to solve problems.

G-7.5 Apply a procedure to draw a top view, front view, and side view of a three-dimensional object.

G-7.6 Apply a procedure to draw an isometric view of a three-dimensional object.

Precalculus

Overview

The academic standards for the precalculus core area establish the process skills and core content for Precalculus, which should provide students with the mathematics skills and conceptual understanding necessary for them to further their mathematical education or to pursue mathematics-related technical careers.

The content of the precalculus standards encompasses characteristics and behaviors of functions, operations on functions, behaviors of polynomial functions and rational functions, behaviors of exponential and logarithmic functions, behaviors of trigonometric functions, and behaviors of conic sections. Teachers, schools, and districts should use the precalculus standards to make decisions concerning the structure and content of Precalculus. Content in this course may go beyond the precalculus standards.

All courses based on the academic standards for precalculus must include instruction using the mathematics process standards, allowing students to engage in problem solving, decision making, critical thinking, and applied learning. Educators must determine the extent to which such courses or individual classes may go beyond these standards. Such decisions will involve choices regarding additional content, activities, and learning strategies and will depend on the objectives of the particular courses or individual classes.

In all courses based on the precalculus standards, hand-held graphing calculators are required for instruction and assessment. Students should learn to use a variety of ways to represent data, to use a variety of mathematical tools such as graph paper, and to use technologies such as graphing calculators to solve problems.

Note: The term *including* appears in parenthetical statements in the high school mathematics indicators to introduce a list of specifics that are intended to clarify and focus the teaching and learning of the particular concept. That is, within these parenthetical including statements are specified the components of the indicator that are critical for the particular core area with regard both to the state assessments and to the management of time in the classroom. While instruction must focus on the entire indicator, educators must be certain to cover the components specified in the parenthetical *including* statements.

HIGH SCHOOL CORE AREA

Precalculus

The mathematical processes provide the framework for teaching, learning, and assessing in all high school mathematics core courses. Instructional programs should be built around these processes.

Standard PC-1: The student will understand and utilize the mathematical processes of problem solving, reasoning and proof, communication, connections, and representation.

Indicators

PC-1.1 Communicate knowledge of algebraic and trigonometric relationships by using mathematical terminology appropriately.
PC-1.2 Connect algebra and trigonometry with other branches of mathematics.
PC-1.3 Apply algebraic methods to solve problems in real-world contexts.
PC-1.4 Judge the reasonableness of mathematical solutions.
PC-1.5 Demonstrate an understanding of algebraic and trigonometric relationships by using a variety of representations (including verbal, graphic, numerical, and symbolic).
PC-1.6 Understand how algebraic and trigonometric relationships can be represented in concrete models, pictorial models, and diagrams.
PC-1.7 Understand how to represent algebraic and trigonometric relationships by using tools such as handheld computing devices, spreadsheets, and computer algebra systems (CASs).

HIGH SCHOOL CORE AREA

Precalculus

Standard PC-2: The student will demonstrate through the mathematical processes an understanding of the characteristics and behaviors of functions and the effect of operations on functions.

Indicators

PC-2.1 Carry out a procedure to graph parent functions (including $y = x^n$, $y = \log_a x$, $y = \ln x$, $y = \frac{1}{x}$, $y = e^x$, $y = a^x$, $y = \sin x$, $y = \cos x$, $y = \tan x$, $y = \csc x$, $y = \sec x$, and $y = \cot x$).

PC-2.2 Carry out a procedure to graph transformations (including $-f(x)$, $a \cdot f(x)$, $f(x) + d$, $f(x - c)$, $f(-x)$, $f(b \cdot x)$, $|f(x)|$, and $f(|x|)$) of parent functions and combinations of transformations.

PC-2.3 Analyze a graph to describe the transformation (including $-f(x)$, $a \cdot f(x)$, $f(x) + d$, $f(x - c)$, $f(-x)$, $f(b \cdot x)$, $|f(x)|$, and $f(|x|)$) of parent functions.

PC-2.4 Carry out procedures to algebraically solve equations involving parent functions or transformations of parent functions (including $y = x^n$, $y = \log_a x$, $y = \ln x$, $y = \frac{1}{x}$, $y = e^x$, $y = a^x$, $y = \sin x$, $y = \cos x$, $y = \tan x$, $y = \csc x$, $y = \sec x$, and $y = \cot x$).

PC-2.5 Analyze graphs, tables, and equations to determine the domain and range of parent functions or transformations of parent functions (including $y = x^n$, $y = \log_a x$, $y = \ln x$, $y = \frac{1}{x}$, $y = e^x$, $y = a^x$, $y = \sin x$, $y = \cos x$, $y = \tan x$, $y = \csc x$, $y = \sec x$, and $y = \cot x$).

PC-2.6 Analyze a function or the symmetry of its graph to determine whether the function is even, odd, or neither.

PC-2.7 Recognize and use connections among significant points of a function (including roots, maximum points, and minimum points), the graph of a function, and the algebraic representation of a function.

PC-2.8 Carry out a procedure to determine whether the inverse of a function exists.

PC-2.9 Carry out a procedure to write a rule for the inverse of a function, if it exists.

HIGH SCHOOL CORE AREA

Precalculus

Standard PC-3: The student will demonstrate through the mathematical processes an understanding of the behaviors of polynomial and rational functions.

Indicators

PC-3.1 Carry out a procedure to graph quadratic and higher-order polynomial functions by analyzing intercepts and end behavior.

PC-3.2 Apply the rational root theorem to determine a set of possible rational roots of a polynomial equation.

PC-3.3 Carry out a procedure to calculate the zeros of polynomial functions when given a set of possible zeros.

PC-3.4 Carry out procedures to determine characteristics of rational functions (including domain, range, intercepts, asymptotes, and discontinuities).

PC-3.5 Analyze given information to write a polynomial function that models a given problem situation.

PC-3.6 Carry out a procedure to solve polynomial equations algebraically.

PC-3.7 Carry out a procedure to solve polynomial equations graphically.

PC-3.8 Carry out a procedure to solve rational equations algebraically.

PC-3.9 Carry out a procedure to solve rational equations graphically.

PC-3.10 Carry out a procedure to solve polynomial inequalities algebraically.

PC-3.11 Carry out a procedure to solve polynomial inequalities graphically.

HIGH SCHOOL CORE AREA

Precalculus

Standard PC-4: The student will demonstrate through the mathematical processes an understanding of the behaviors of exponential and logarithmic functions.

Indicators

PC-4.1 Carry out a procedure to graph exponential functions by analyzing intercepts and end behavior.

PC-4.2 Carry out a procedure to graph logarithmic functions by analyzing intercepts and end behavior.

PC-4.3 Carry out procedures to determine characteristics of exponential functions (including domain, range, intercepts, and asymptotes).

PC-4.4 Carry out procedures to determine characteristics of logarithmic functions (including domain, range, intercepts, and asymptotes).

PC-4.5 Apply the laws of exponents to solve problems involving rational exponents.

PC-4.6 Analyze given information to write an exponential function that models a given problem situation.

PC-4.7 Apply the laws of logarithms to solve problems.

PC-4.8 Carry out a procedure to solve exponential equations algebraically.

PC-4.9 Carry out a procedure to solve exponential equations graphically.

PC-4.10 Carry out a procedure to solve logarithmic equations algebraically.

PC-4.11 Carry out a procedure to solve logarithmic equations graphically.

HIGH SCHOOL CORE AREA

Precalculus

Standard PC-5: The student will demonstrate through the mathematical processes an understanding of the behaviors of trigonometric functions.

Indicators

PC-5.1 Understand how angles are measured in either degrees or radians.
PC-5.2 Carry out a procedure to convert between degree and radian measures.
PC-5.3 Carry out a procedure to plot points in the polar coordinate system.
PC-5.4 Carry out a procedure to graph trigonometric functions by analyzing intercepts, periodic behavior, and graphs of reciprocal functions.
PC-5.5 Carry out procedures to determine the characteristics of trigonometric functions (including domain, range, intercepts, and asymptotes).
PC-5.6 Apply a procedure to evaluate trigonometric expressions.
PC-5.7 Analyze given information to write a trigonometric function that models a given problem situation involving periodic phenomena.
PC-5.8 Analyze given information to write a trigonometric equation that models a given problem situation involving right triangles.
PC-5.9 Carry out a procedure to calculate the area of a triangle when given the lengths of two sides and the measure of the included angle.
PC-5.10 Carry out a procedure to solve trigonometric equations algebraically.
PC-5.11 Carry out a procedure to solve trigonometric equations graphically.
PC-5.12 Apply the laws of sines and cosines to solve problems.
PC-5.13 Apply a procedure to graph the inverse functions of sine, cosine, and tangent.
PC-5.14 Apply trigonometric relationships (including reciprocal identities; Pythagorean identities; even and odd identities; addition and subtraction formulas of sine, cosine, and tangent; and double angle formulas) to verify other trigonometric identities.
PC-5.15 Carry out a procedure to compute the slope of a line when given the angle of inclination of the line.

HIGH SCHOOL CORE AREA

Precalculus

Standard PC-6: The student will demonstrate through the mathematical processes an understanding of the behavior of conic sections both geometrically and algebraically.

Indicators

PC-6.1 Carry out a procedure to graph the circle whose equation is the form $(x-h)^2+(y-k)^2=r^2$.

PC-6.2 Analyze given information about the center and the radius or the center and the diameter to write an equation of a circle.

PC-6.3 Apply a procedure to calculate the coordinates of points where a line intersects a circle.

PC-6.4 Carry out a procedure to graph the ellipse whose equation is the form $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$.

PC-6.5 Carry out a procedure to graph the hyperbola whose equation is the form $\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$.

PC-6.6 Carry out a procedure to graph the parabola whose equation is the form $y-k=a(x-h)^2$.

Data Analysis and Probability

Overview

The academic standards for the data analysis and probability core area establish the process skills and core content for Probability and Statistics and Mathematics for the Technologies 4, which should provide students with the mathematics skills and conceptual understanding necessary for them to further their mathematical education or to pursue mathematics-related technical careers.

The content of the data analysis and probability standards encompasses design of a statistical study; collection, organization, display, and interpretation of data; basic statistical methods of analyzing data; and basic concepts of probability. Teachers, schools, and districts should use the data analysis and probability standards to make decisions concerning the structure and content of Probability and Statistics and Mathematics for the Technologies 4. Content in these two courses may go beyond the data analysis and probability standards.

All courses based on the academic standards for data analysis and probability must include instruction using the mathematics process standards, allowing students to engage in problem solving, decision making, critical thinking, and applied learning. Educators must determine the extent to which such courses or individual classes may go beyond these standards. Such decisions will involve choices regarding additional content, activities, and learning strategies and will depend on the objectives of the particular courses or individual classes.

In all courses based on the data analysis and probability standards, hand-held graphing calculators are required for instruction and assessment. Students should learn to use a variety of ways to represent data, to use a variety of mathematical tools such as graph paper, and to use technologies such as graphing calculators to solve problems.

Note: The term *including* appears in parenthetical statements in the high school mathematics indicators to introduce a list of specifics that are intended to clarify and focus the teaching and learning of the particular concept. That is, within these parenthetical including statements are specified the components of the indicator that are critical for the particular core area with regard both to the state assessments and to the management of time in the classroom. While instruction must focus on the entire indicator, educators must be certain to cover the components specified in the parenthetical *including* statements.

HIGH SCHOOL CORE AREA

Data Analysis and Probability

The mathematical processes provide the framework for teaching, learning, and assessing in all high school mathematics core courses. Instructional programs should be built around these processes.

Standard DA-1: The student will understand and utilize the mathematical processes of problem solving, reasoning and proof, communication, connections, and representation.

Indicators

DA-1.1 Execute procedures to conduct simple probability experiments and collect data by using manipulatives (including spinners, dice, cards, and coins).

DA-1.2 Execute procedures to find measures of probability and statistics by using tools such as handheld computing devices, spreadsheets, and statistical software.

DA-1.3 Execute procedures to conduct a simulation by using random number tables and/or technology (including handheld computing devices and computers).

DA-1.4 Design and conduct a statistical research project and produce a report that summarizes the findings.

DA-1.5 Apply the principles of probability and statistics to solve problems in real-world contexts.

DA-1.6 Communicate a knowledge of data analysis and probability by using mathematical terminology appropriately.

DA-1.7 Judge the reasonableness of mathematical solutions on the basis of the source of the data, the design of the study, the way the data are displayed, and the way the data are analyzed.

DA-1.8 Compare data sets by using graphs and summary statistics.

HIGH SCHOOL CORE AREA

Data Analysis and Probability

Standard DA-2: The student will demonstrate through the mathematical processes an understanding of the design of a statistical study.

Indicators

DA-2.1 Classify a data-collection procedure as a survey, an observational study, or a controlled experiment.

DA-2.2 Compare various random sampling techniques (including simple, stratified, cluster, and systematic).

DA-2.3 Analyze a data-collection procedure to classify the technique used as either simple cluster, systematic, or convenience sampling.

DA-2.4 Critique data-collection methods and describe how bias can be controlled.

DA-2.5 Judge which of two or more possible experimental designs will best answer a given research question.

DA-2.6 Generate a research question and design a statistical study to answer a given research question.

HIGH SCHOOL CORE AREA

Data Analysis and Probability

Standard DA-3: The student will demonstrate through the mathematical processes an understanding of the methodology for collecting, organizing, displaying, and interpreting data.

Indicators

DA-3.1 Use manipulatives, random number tables, and technology to collect data and conduct experiments and simulations.

DA-3.2 Organize and interpret data by using pictographs, bar graphs, pie charts, dot plots, histograms, time-series plots, stem-and-leaf plots, box-and-whiskers plots, and scatterplots.

DA-3.3 Select appropriate graphic display(s) from among pictographs, bar graphs, pie charts, dot plots, histograms, time-series plots, stem-and-leaf plots, box-and-whiskers plots, and scatterplots when given a data set or problem situation.

DA-3.4 Represent frequency distributions by using displays such as categorical frequency distributions/Pareto charts, histograms, frequency polygons, and cumulative frequency distributions/ogives.

DA-3.5 Classify a scatterplot by shape (including linear, quadratic, and exponential).

DA-3.6 Classify graphically and analytically the correlation between two variables as either positive, negative, or zero.

DA-3.7 Carry out a procedure to determine an equation of a trend line for a scatterplot exhibiting a linear pattern by using visual approximation.

DA-3.8 Carry out a procedure using technology to determine a line of best fit for a scatterplot exhibiting a linear pattern.

DA-3.9 Explain the meaning of the correlation coefficient r.

DA-3.10 Use interpolation or extrapolation to predict values based on the relationship between two variables.

HIGH SCHOOL CORE AREA

Data Analysis and Probability

Standard DA-4: The student will demonstrate through the mathematical processes an understanding of basic statistical methods of analyzing data.

Indicators

DA-4.1 Classify a variable as either a statistic or a parameter.
DA-4.2 Compare descriptive and inferential statistics.
DA-4.3 Classify a variable as either discrete or continuous and as either categorical or quantitative.
DA-4.4 Use procedures and/or technology to find measures of central tendency (mean, median, and mode) for given data.
DA-4.5 Predict the effect of transformations of data on measures of central tendency, variability, and the shape of the distribution.
DA-4.6 Use procedures and/or technology to find measures of spread (range, variance, standard deviation, and interquartile range) and outliers for given data.
DA-4.7 Use procedures and/or technology to find measures of position (including median, quartiles, percentiles, and standard scores) for given data.
DA-4.8 Classify a distribution as either symmetric, positively skewed, or negatively skewed.
DA-4.9 Explain the significance of the shape of a distribution.
DA-4.10 Use a knowledge of the empirical rule to solve problems involving data that are distributed normally.
DA-4.11 Use control charts to determine whether a process is in control.

HIGH SCHOOL CORE AREA

Data Analysis and Probability

Standard DA-5: The student will demonstrate through the mathematical processes an understanding of the basic concepts of probability.

Indicators

DA-5.1 Construct a sample space for an experiment and represent it as a list, chart, picture, or tree diagram.
DA-5.2 Use counting techniques to determine the number of possible outcomes for an event.
DA-5.3 Classify events as either dependent or independent.
DA-5.4 Categorize two events either as mutually exclusive or as not mutually exclusive of one another.
DA-5.5 Use the concept of complementary sets to compute probabilities.
DA-5.6 Use the binomial probability distribution to solve problems.
DA-5.7 Carry out a procedure to compute simple probabilities and compound probabilities (including conditional probabilities).
DA-5.8 Use a procedure to find geometric probability in real-world contexts.
DA-5.9 Compare theoretical and experimental probabilities.
DA-5.10 Construct and compare theoretical and experimental probability distributions.
DA-5.11 Use procedures to find the expected value of discrete random variables and construct meaning within contexts.
DA-5.12 Understand the law of large numbers.
DA-5.13 Carry out a procedure to compute conditional probability by using two-way tables.

APPENDIX

Revised Bloom's Taxonomy

In 1956, Benjamin Bloom and his colleagues published the *Taxonomy of Educational Objectives: The Classification of Educational Goals*, a groundbreaking book that classified educational goals according to the cognitive processes that learners must use in order to attain those goals. The work, which was enthusiastically received, was utilized by teachers to analyze learning in the classroom for nearly fifty years.

However, research during that time span generated new ideas and information about how learners learn and how teachers teach. Education practice is very different today. Even the measurement of achievement has changed: teachers now live in a standards-based world defined by state accountability systems.

In order to reflect the new data and insights about teaching and learning that the past forty-five years of research have yielded—and to refocus educators' attention on the value of the original Bloom's taxonomy—Lorin Anderson and David Krathwohl led a team of colleagues in revising and enhancing that system to make it more usable for aligning standards, instruction, and assessment in today's schools. The results of their work were published in 2001 as *A Taxonomy for Learning, Teaching, and Assessing: A Revision of Bloom's Taxonomy of Educational Objectives* (New York: Allyn and Bacon)—a book that is important to educators because it provides the common understanding of expectations that is critical for improving student achievement in all subjects.

The revised taxonomy is two-dimensional, identifying both the kind of knowledge to be learned (knowledge dimension) and the kind of learning expected from students (cognitive processes) to help teachers and administrators improve alignment and rigor in the classroom. This taxonomy will assist educators to improve instruction, to ensure that their lessons and assessments are aligned with one another and with the state standards, that their lessons are cognitively rich, and that instructional opportunities are not missed.

Mathematics goes well beyond simple recognition and recall and the memorization of facts that many people mistake for the core of mathematics. The verbs in the indicators of the 2007 mathematics academic standards are subcategories of the six cognitive processes described in the revised Bloom's taxonomy. The verbs are intentionally selected to be appropriate when teaching the particular content in each indicator. For example, one might *compare* attributes of congruent shapes or *classify* shapes as congruent. Both of these are included in the cognitive process dimension *understand*, which has five other processes: *interpreting, exemplifying, summarizing, inferring*, and *explaining*. All seven subcategories are important aspects of *understanding* and should be part of the learning process for that indicator when they are appropriate for the content. In addition, cognitive process categories lower on the taxonomy may need to be addressed in order to reach the next level. For example, students need to *recognize* and *recall* attributes of shapes to *compare* them. State assessments such as the PACT might address any of the

subcategories in a particular cognitive category or categories lower on the taxonomy as appropriate to the content. Beginning with these revised mathematics standards, descriptions of the kinds of learning required in South Carolina standards will be drawn directly from the revised Bloom's taxonomy.

Tables 1 and 2 below are reproduced from Anderson and Krathwohl's *Taxonomy for Learning, Teaching, and Assessing*, pages 46 and 67, respectively. Table 3, "A Taxonomy for Teaching, Learning, and Assessing," describes both dimensions of the taxonomy: the categories and subcategories of knowledge described in table 1 and the cognitive processes described in table 2. This matrix is provided as a template for teachers to use in analyzing their instruction as they seek to align standards, units/lessons/activities, and assessments. Examples and more information about specific uses of the matrix can be found in the *Taxonomy for Learning, Teaching and Assessing.*

Table 1: The Knowledge Dimension

Major Types and Subtypes		Examples
A. Factual Knowledge—The basic elements students must know to be acquainted with a discipline or solve problems in it		
Aa.	Knowledge of terminology	Technical vocabulary, musical symbols
Ab.	Knowledge of specific details and elements	Major natural resources, reliable sources of information
B. Conceptual Knowledge—The interrelationships among the basic elements within a larger structure that enable them to function together		
Ba.	Knowledge of classifications and categories	Periods of geological time, forms of business ownership
Bb.	Knowledge of principles and generalizations	Pythagorean theorem, law of supply and demand
Bc.	Knowledge of theories, models, and structures	Theory of evolution, structure of Congress
C. Procedural Knowledge—How to do something, methods and inquiry, and criteria for using skills, algorithms, techniques, and methods		
Ca.	Knowledge of subject-specific skills and algorithms	Skills used in painting with watercolors, whole-number division algorithm
Cb.	Knowledge of subject-specific techniques and methods	Interviewing techniques, scientific method
Cc.	Knowledge of criteria for determining when to use appropriate procedures	Criteria used to determine when to apply a procedure involving Newton's second law, criteria used to judge the feasibility of using a particular method to estimate business costs
D. Metacognitive Knowledge—Knowledge of cognition in general as well as awareness and knowledge of one's own cognition		
Da.	Strategic knowledge	Knowledge of outlining as a means of capturing the structure of a unit of subject matter in a textbook, knowledge of the use of heuristics
Db.	Knowledge about cognitive tasks, including appropriate contextual and conditional knowledge	Knowledge of the types of tests particular teachers administer, knowledge of the cognitive demands of different tasks
Dc.	Self-knowledge	Knowledge that critiquing essays is a personal strength, whereas writing essays is a personal weakness; awareness of one's own knowledge level

Table 2: The Cognitive Process Dimension

Categories & Cognitive Processes	Alternative Names	Definitions and Examples
1. REMEMBER—Retrieve relevant knowledge from long-term memory		
1.1 Recognizing	Identifying	Locating knowledge in long-term memory that is consistent with presented material (e.g., Recognize the dates of important events in United States history)
1.2 Recalling	Retrieving	Retrieving relevant knowledge from long-term memory (e.g., Recall the dates of important events in United States history)
2. UNDERSTAND—Construct meaning from instructional messages, including oral, written, and graphic communication		
2.1 Interpreting	Clarifying, paraphrasing, representing, translating	Changing from one form of representation (e.g., numerical) to another (e.g., verbal) (e.g., Paraphrase important speeches and documents)
2.2 Exemplifying	Illustrating, instantiating	Finding a specific example or illustration of a concept or principle (e.g., Give examples of various artistic painting styles)
2.3 Classifying	Categorizing, subsuming	Determining that something belongs to a category (e.g., Classify observed or described cases of mental disorders)
2.4 Summarizing	Abstracting, generalizing	Abstracting a general theme or major point(s) (e.g., Write a short summary of events portrayed on a videotape)
2.5 Inferring	Concluding, extrapolating, interpolating, predicting	Drawing a logical conclusion from presented information (e.g., In learning a foreign language, infer grammatical principles from examples)
2.6 Comparing	Contrasting, mapping, matching	Detecting correspondences between two ideas, objects, and the like (e.g., Compare historical events to contemporary situations)
2.7 Explaining	Constructing models	Constructing a cause-and-effect model of a system (e.g., Explain the causes of important 18th Century events in France)
3. APPLY—Carry out or use a procedure in a given situation		
3.1 Executing	Carrying out	Applying a procedure to a familiar task (e.g., Divide one whole number by another whole number, both with multiple digits)
3.2 Implementing	Using	Applying a procedure to an unfamiliar task (e.g., Use Newton's Second Law in situations in which it is appropriate)

Table 2: The Cognitive Process Dimension

Categories & Cognitive Processes	Alternative Names	Definitions and Examples
4. ANALYZE—Break material into its constituent parts and determine how the parts relate to one another and to an overall structure or purpose		
4.1 Differentiating	Discriminating, distinguishing, focusing, selecting	Distinguishing relevant from irrelevant parts or important from unimportant parts of presented material (e.g., Distinguish between relevant and irrelevant numbers in a mathematical word problem)
4.2 Organizing	Finding coherence, integrating, outlining, parsing, structuring	Determining how elements fit or function within a structure (e.g., Structure evidence in a historical description into evidence for and against a particular historical explanation)
4.3 Attributing	Deconstructing	Determine a point of view, bias, values, or intent underlying presented material (e.g., Determine the point of view of the author of an essay in terms of his or her political perspective)
5. EVALUATE—Make judgments based on criteria and standards		
5.1 Checking	Coordinating, detecting, monitoring, testing	Detecting inconsistencies or fallacies within a process or product; determining whether a process or product has internal consistency; detecting the effectiveness of a procedure as it is being implemented (e.g., Determine if a scientist's conclusions follow from observed data)
5.2 Critiquing	Judging	Detecting inconsistencies between a product and external criteria, determining whether a product has external consistency; detecting the appropriateness of a procedure for a given problem (e.g., Judge which of two methods is the best way to solve a given problem)
6. CREATE—Put elements together to form a coherent or functional whole; reorganize elements into a new pattern or structure		
6.1 Generating	Hypothesizing	Coming up with alternative hypotheses based on criteria (e.g., Generate hypotheses to account for an observed phenomenon)
6.2 Planning	Designing	Devising a procedure for accomplishing some task (e.g., Plan a research paper on a given historical topic)
6.3 Producing	Constructing	Inventing a product (e.g., Build habitats for a specific purpose)

The revised standards for kindergarten through grade eight consist of six

Table 3: A Taxonomy for Teaching, Learning, and Assessing

THE KNOWLEDGE DIMENSION	THE COGNITIVE PROCESS DIMENSION					
	1. Remember—Retrieve relevant knowledge from long-term memory 1.1 Recognizing 1.2 Recalling	**2. Understand**—Construct meaning from instructional messages, including oral, written, and graphic communication 2.1 Interpreting 2.2 Exemplifying 2.3 Classifying 2.4 Summarizing 2.5 Inferring 2.6 Comparing 2.7 Explaining	**3. Apply**—Carry out or use a procedure in a given situation 3.1 Executing 3.2 Implementing	**4. Analyze**—Break material into its constituent parts and determine how the parts relate to one another and to an overall structure or purpose 4.1 Differentiating 4.2 Organizing 4.3 Attributing	**5. Evaluate**—Make judgments based on criteria and standards 5.1 Checking 5.2 Critiquing	**6. Create**—Put elements together to form a coherent or functional whole; reorganize elements into a new pattern or structure 6.1 Generating 6.2 Planning 6.3 Producing
A. Factual Knowledge—The basic elements that students must know to be acquainted with a discipline or solve problems in it AA. Knowledge of terminology AB. Knowledge of specific details and elements						
B. Conceptual Knowledge—The interrelationships among the basic elements within a larger structure that enable them to function together BA. Knowledge of classifications and categories BB. Knowledge of principles and generalizations BC. Knowledge of theories, models, and structures						
C. Procedural Knowledge—How to do something, methods of inquiry, and criteria for using skills, algorithms, techniques, and methods CA. Knowledge of subject-specific skills and algorithms CB. Knowledge of subject-specific techniques and methods CC. Knowledge of criteria for determining when to use appropriate procedures						
D. Metacognitive Knowledge—Knowledge of cognition in general as well as awareness of one's own cognition DA. Strategic knowledge DB. Knowledge about cognitive tasks (including appropriate contextual and conditional knowledge DC. Self-knowledge						