

20.5 Multiplicative Inverses

Real #'s

$$\frac{2}{1} \cdot \frac{1}{2} = 1$$

Complex #'s

$$a + bi \cdot \frac{1}{a + bi} = 1$$

We have to rationalize the denominator though!

ex) $3 - i \cdot \frac{1}{3 - i} = 1$

To find the **Multiplicative Inverse** of $a + bi$:

Quick Trick = $\frac{a - bi}{a^2 + b^2}$ OR Multiply by the Conjugate

ex) Find the mult. inv of $3 - i$ and simplify if possible

$$3 - i \left(\frac{1}{3 - i} \right) \rightarrow \frac{3 + i}{3^2 + 1^2} = \frac{3 + i}{10}$$

$$\frac{1}{(3 - i)(3 + i)} = \frac{3 + i}{9 + 3i - 3i - i^2}$$

$$9 + 1$$

$$\frac{3 + i}{10}$$

Find the multiplicative inverse of $2 + 4i$ and simplify if possible.

$$2+4i \left(\frac{1}{2+4i} \right)$$

$$\frac{2-4i}{2^2+4^2} = \frac{\cancel{2}-4i}{\cancel{2} \times 10} = \frac{\cancel{2}(1-2i)}{\cancel{2} \times 10}$$

$$(2+4i) \left(\frac{1-2i}{10} \right)$$

$$\frac{\cancel{2}-\cancel{4}i + 4i - 8i^2}{10}$$

$$\frac{2+8}{10} = \frac{10}{10} = 1$$

Find the multiplicative inverse of $8 - 4i$ and simplify if possible.

$$\frac{2+i}{20}$$

$$7+3i$$

$$\frac{7-3i}{49+9} = \frac{7-3i}{58}$$

Prove that $1 + i$ and $\frac{1 - i}{2}$ are multiplicative inverses

$$\frac{(1+i)(1-i)}{2}$$

$$\frac{1+1}{2} = \frac{2}{2} = 1 \quad \checkmark$$

Division of Complex Numbers:

Divide $8 + i$ by $2 - i$

STEPS:

1. Put them on top of each other
2. Multiply by the conjugate
3. Simplify

$$\frac{(8+i)(2+i)}{(2-i)(2+i)}$$

$$\frac{16 + 8i + 2i + i^2}{4 + 1} = \frac{15 + 10i}{5} = \frac{5(3+2i)}{5}$$

Check

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$$(3+2i)(2-i) = 6 - 3i + 4i - 2i^2$$

✓ $8+i$

Divide and Check:

$$(3+12i) \div (4 - i)$$

Homework: pg 940 # 1,3,5,11,13,24,25,27,30,34